Royal Society Te Apārangi

Expert Advisory Panel

Pāngarau Mathematics and Tauanga Statistics in Aotearoa New Zealand

Advice on refreshing the English-medium Mathematics and Statistics learning area of the New Zealand Curriculum

Mahuru September 2021

ROYAL SOCIETY TE APĀRANGI

Ngā Mihi

Acknowledgements

Panel members were Distinguished Professor Gaven Martin (chair), Professor Glenda Anthony, Professor Jennifer Brown, Dr Michelle Dalrymple, Associate Professor Fiona Ell, Associate Professor Sina Greenwood, Associate Professor Joanna Higgins, Associate Professor Jodie Hunter, Associate Professor Rua Murray, Associate Professor Matthew Roskruge, Associate Professor Tony Trinick and Associate Professor Caroline Yoon.

The Panel would like to thank the New Zealand Mathematical Society and the New Zealand Statistical Society for valuable commentary on an earlier draft of this report.

The report was independently reviewed by Associate Professor Robin Averill, Dr Jane McChesney, Dr Tanya Evans, and Christine Franklin (University of Georgia, USA, Emerita Statistics Faculty).

The Panel's work was supported by Royal Society Te Apārangi staff members Dr Andrew Cleland FRSNZ, Dr Roger Ridley, and Dr Heidi de Ronde.

Whakarāpopototanga

Executive Summary

The Royal Society Te Apārangi Expert Advisory Panel on Mathematics and Statistics convened between January and June 2021 with a brief to provide advice to the Ministry of Education on the English-medium Mathematics and Statistics curriculum in Aotearoa New Zealand. The Ministry sought the Panel's advice as part of a wider programme of work on refreshing the Mathematics and Statistics curriculum. This included analysis of data from national and international benchmark testing and monitoring of student assessment, and seeking advice from profession-focused reference groups along with other interested parties. The Panel comprised mathematicians, statisticians, economists, and mathematics educators, representing diverse experiences of the mathematics and statistics curriculum and mathematics and statistics teaching and learning in schooling.

The Terms of Reference for the Panel centred on defining the mathematics and statistics that learners need to know and when they should know it by; consideration of what 'enough' mathematics and statistics might be for critical engagement with society; the relationship between numeracy and mathematics; what the essential 'big ideas' of mathematics and statistics in schooling should be; and the Panel's views on progress and assessment.

The Panel quickly discovered that these elements of curriculum design were less material to ākonga student progress and achievement in Aotearoa New Zealand than the many complex factors that surround their engagement with mathematics and statistics in schools. The Panel became concerned that suggesting curriculum changes alone, without acknowledging and addressing key aspects of the context in which the curriculum is being taught and learned, might support the conclusion that simply changing the official curriculum and its syllabus would leverage transformation in ākonga student outcomes. The Panel does not believe it will.

An up-to-date, relevant, and appropriately challenging curriculum is a necessary foundation for improved achievement and equity of outcomes in mathematics and statistics for ākonga students. However, the Panel believes such changes to the curriculum are unlikely to lead to beneficial change without considering the interdependencies among the curriculum's content and structure, teaching approaches, resourcing, and schools and communities. Hence, this report situates the Panel's commentary within the wider frame of mathematics and statistics education in English-medium compulsory schooling, focusing on years 0–10 and, more broadly, teases out some of the most important relationships and dependencies between the written curriculum and student learning experiences as the Panel sees them.

The report includes 14 recommendations to the Ministry of Education centred on four themes:

- **Slippage:** This theme describes how year on year many of our ākonga students fall away from the trajectory described by the current Mathematics and Statistics curriculum. It leads to outcomes such as fewer than one in ten 14-year-olds are in classes where the majority of their peers are working at the appropriate curriculum level. Indeed, if most ākonga students achieved the current curriculum levels in the suggested timeframe, they would be well positioned to succeed in mathematics and statistics, and use mathematics effectively in their daily lives.
- **Teacher disciplinary and pedagogical knowledge:** This theme refers to teacher knowledge of mathematics and statistics itself and the teaching of mathematics and statistics. In Aotearoa New Zealand very few primary teachers have specialised in mathematics or mathematics teaching and only 14% of Year 5 teachers have specialised. Yet, studies have shown that teacher knowledge 'well beyond the student level' is essential to be able to answer questions meaningfully, make connections, make the most of the teachable moments, and plan how to implement the curriculum. Teachers deserve support and professional learning in mathematics and statistics so they can more easily, effectively, and confidently teach ākonga students.
- **Leadership:** This theme recognises the need for a more centralised approach to support coherence and clarity around teaching and learning in mathematics and statistics education. This includes fixing the problem of significant numbers of under-prepared teachers in mathematics and statistics, and simplifying the selection and delivery of suites of excellent supporting material written by content-matter and pedagogical experts – it currently falls on teachers to choose within a vast array of available material. Leadership is also necessary to drive professional development and classroom support in hard-to-staff schools and disciplines, and to navigate change in fast-moving areas where resources may date quickly.
- **Inequity:** The evidence is that our current mathematics and statistics education system as a whole perpetuates, indeed grows, inequity. While this is a general characteristic, it must be addressed specifically within the context of mathematics and statistics teaching and learning. The Panel does not expect that all ākonga students will achieve equally, or study mathematics and statistics with a passion, but we do expect that all ākonga students will have an opportunity to develop a positive and productive relationship with mathematics and statistics in order to develop the skills and knowledge that will realise the opportunities and life outcomes such engagement delivers.

The current state of mathematics and statistics education in schools is the product of a deeply complex system working at multiple levels. Teachers are at the heart of our education system; they have met all the requirements placed on them to be fully qualified and cannot be blamed for how the system works. It is the Panel's view that if Aotearoa New Zealand wants equitable outcomes and enhanced achievement for our ākonga students, we need substantial, considered investment in mathematics and statistics education, and change at virtually all levels of the education system.

Ngā Ihirangi

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Kupu Whakataki

Introduction

It is critically important that all Aotearoa New Zealand's ākonga students learn pāngarau mathematics and tauanga statistics thoroughly and well. This is not the case at the moment. Our ākonga students are underserved by our current system, as is our nation.

For thousands of years, societies have handed down mathematical practices to the next generation. We teach mathematics because it provides a tool for understanding the world around us. Mathematics also contributes to the development of important intellectual skills such as problem solving, logic, deductive and inductive reasoning, abstraction, spatial reasoning, creative thinking, and communication, as well as ways to think about data, information, and living with uncertainty. Beyond being an essential skill for both employees and entrepreneurs, basic numeracy impacts almost every aspect of our lives. For these reasons, early achievement in mathematics is a strong predictor – even more so than reading skills – of later academic achievement, mobility, economic success, and civic participation.[1]

However, concerns about the learning of mathematics and statistics are positioned within a paradox. While there is clear social concern over students' under-performance in mathematics and statistics, it seems to be acceptable to admit to being 'no good' at mathematics and statistics, and to regard mathematics and statistics as for 'other people' rather than being necessary and important for all. This paradox does not serve us well as a society.

We are in an era of change in the mathematical sciences. Statistical modelling is now commonplace in all the sciences and humanities, with data and its analysis underpinning the evidence that is used widely in formulating scientific explanations, understanding social, economic, and environmental change, and modelling the impact of policy decisions. The ubiquity of digital technologies and global connectivity mean that in day-to-day life we are surrounded by devices that are underpinned by sophisticated mathematics. These changes are happening apace, and our national curriculum needs to be responsive and agile. There is little doubt that the science and technology of the 21st century (and beyond) will be in large part about the manipulation, handling, analysis, and interpretation of physical and computational data, as well as computational thinking and the rise of artificial intelligence in decision making – in all walks of life. Mathematics and statistics are more relevant and important than ever, and Aotearoa New Zealand needs citizens with well-developed mathematical and statistical skills for the positive work and life outcomes they confer, and the opportunities they present to engage effectively in our democracy.

The Panel believes that **all children in Aotearoa New Zealand, irrespective of their individual life circumstances, must be given opportunities to develop sufficient competence and confidence in pāngarau mathematics and tauanga statistics that enables them to lead rewarding and fulfilling lives.** This is the operating assumption on which the Panel has based its recommendations, and what the Panel intends its recommendations to achieve. **Given the right circumstances, everyone can learn mathematics and statistics, and a significant proportion can excel.**

[1] Holenstein, M., Bruckmaier, G. & Grob, A. Transfer effects of mathematical literacy: An integrative longitudinal study. *Eur J Psychol Educ* (2020). https://doi.org/10.1007/s10212-020-00491-4

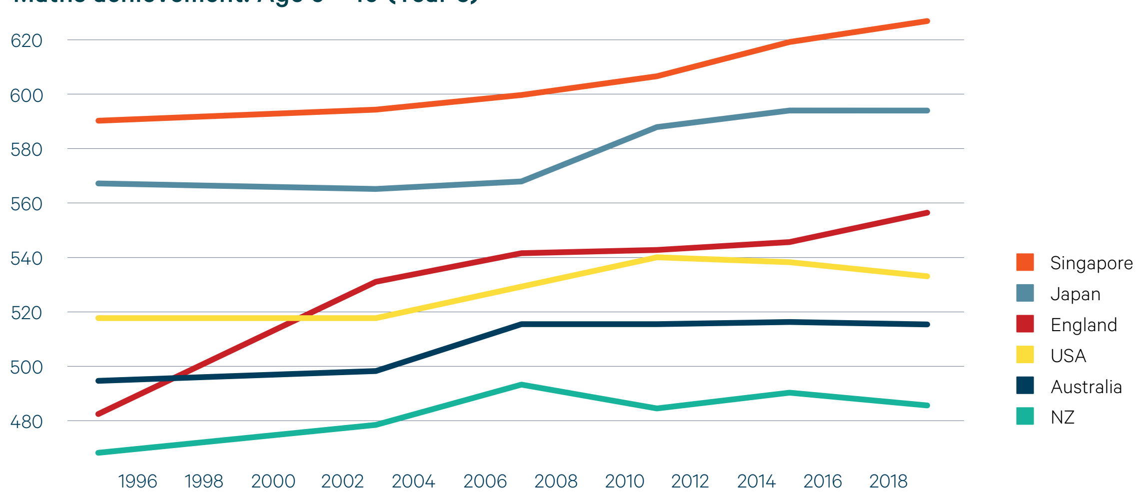
Against these aspirations is Aotearoa New Zealand's actual performance against its internal expectations and international benchmarks.[2] While there are some positive signs – 81% of Year 4 students are achieving at Level 2 of the curriculum (NMSSA) – overall the situation is troubling: nearly 25% of mothers report that their four-and-a-half-year-old children cannot count to 10 (GuiNZ); fewer than one-third of Year 5 students are in classes where teachers report that the majority were working at the expected level (Level 3) of the curriculum – a big slip from Year 4 and indicating a lack of progression; 44% of Year 5 students are low or very low achievers (TIMSS 2018) and the proportion of very low achievers is over twice the international average (17% compared with 8%); 55% of Year 8 students achieve below Level 4 curriculum expectations (NMSSA); fewer than one-tenth of Year 9 students are in classes where the majority were working at the appropriate level (Level 5) of the curriculum; and 47% of Year 9 students were low or very low achievers (TIMSS). Overall, New Zealand has a growing number of very low achievers.

While there are high and low achievers in all schools, Aotearoa New Zealand had higher achievement on average among higher socioeconomic groupings. The gap in achievement between higher and lower socioeconomic groups is very large – larger than most other nations, with the NMSSA estimating the difference at more than two years of schooling. Māori and Pacific learners were over-represented among lower achievers and under-represented among higher achievers.

The graphs below are taken from TIMSS data and show low student achievement, consistently at or below OECD averages (around 500), along with a selection of other countries.

FIGURE 1

Maths achievement: Age 9 – 10 (Year 5)

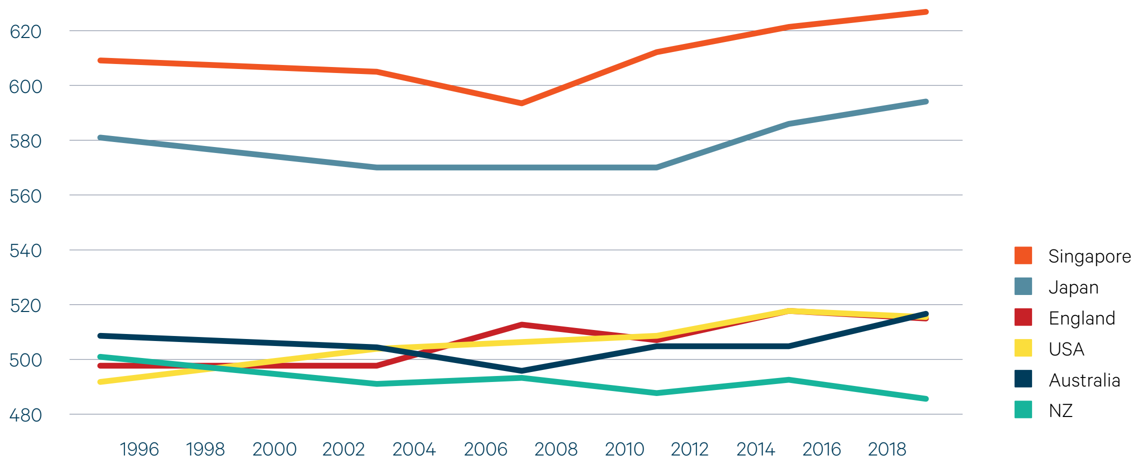


Source: Trends in International Mathematics and Science Study (TIMSS)

[2] *Growing Up in New Zealand (GUiNZ)* study at age 4.5; *Trends in International Mathematics and Science Study (TIMSS)*, conducted with Year 5 and Year 9 students, their teachers, and their schools; *National Monitoring Study of Student Achievement (NMSSA)*, conducted with Year 4 and Year 8 students, their teachers, and their schools; *The electronic assessment tool for teaching and learning (e-asTTle)*, developed primarily for learners in Years 5 to 10; *Programme for International Student Assessment (PISA)*, conducted with 15-year-old students' Data from the *National Certificate of Educational Achievement (NCEA)*.

FIGURE 2

Maths achievement: Age 13 – 14 (Year 9)

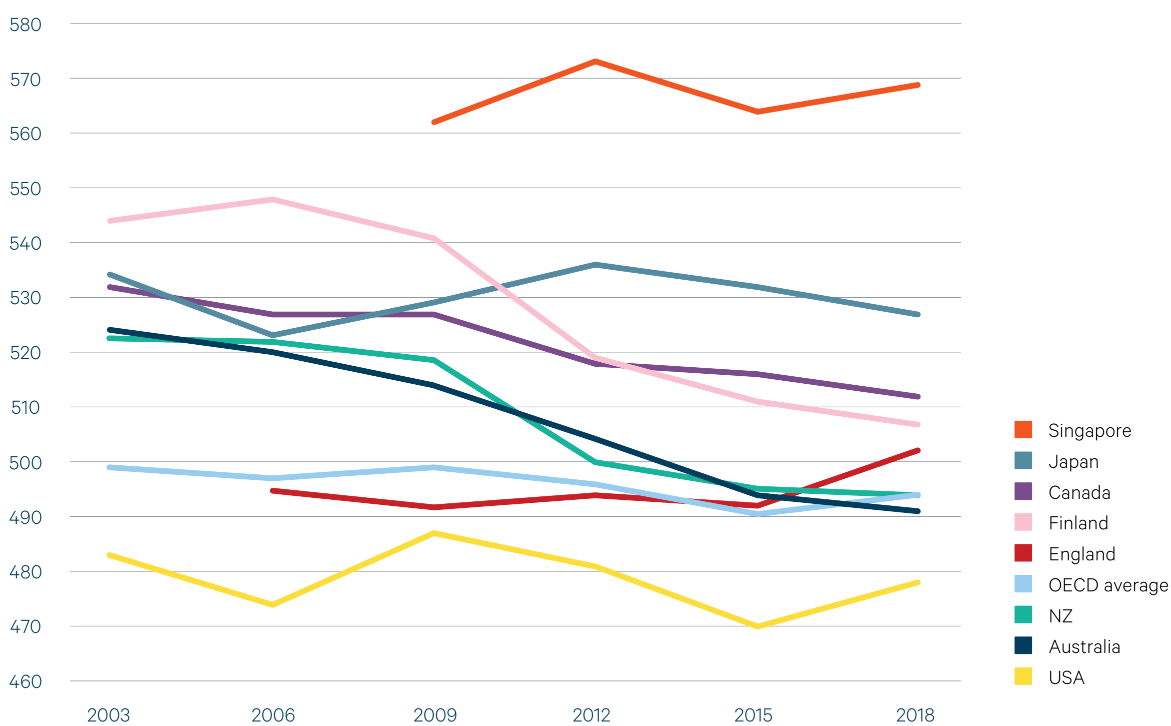


Source: Trends in International Mathematics and Science Study (TIMSS)

Similarly, the results from the PISA benchmarks from 2003 are shown below. Of particular note is the substantial decline in Finland (39pts), Australia (33pts) and New Zealand (29pts) against the relative stability of the OECD average (5pts). These countries appear to be grappling with similar issues.

FIGURE 3

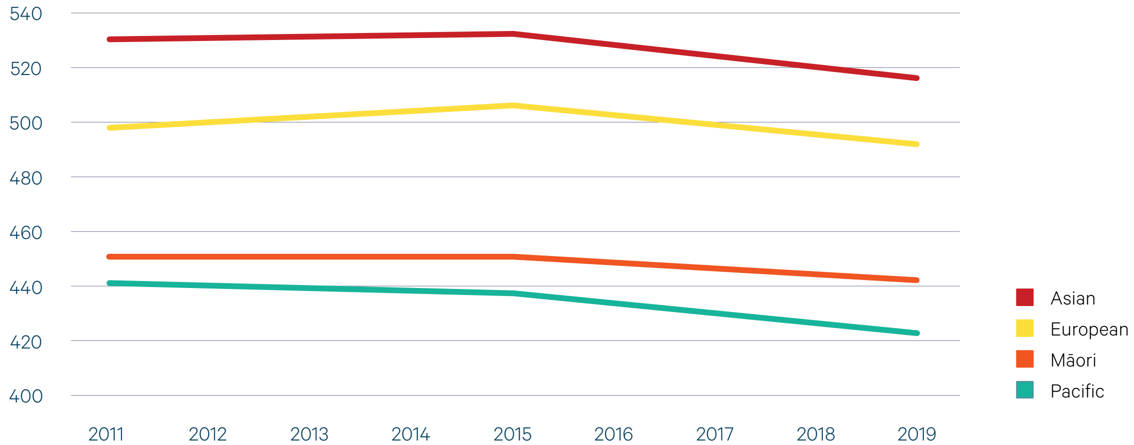
PISA benchmarks from 2003



The following graphs from TIMSS data show that our current system perpetuates, indeed exacerbates, inequity of outcomes. This is also supported by the NMSSA data.

FIGURE 4

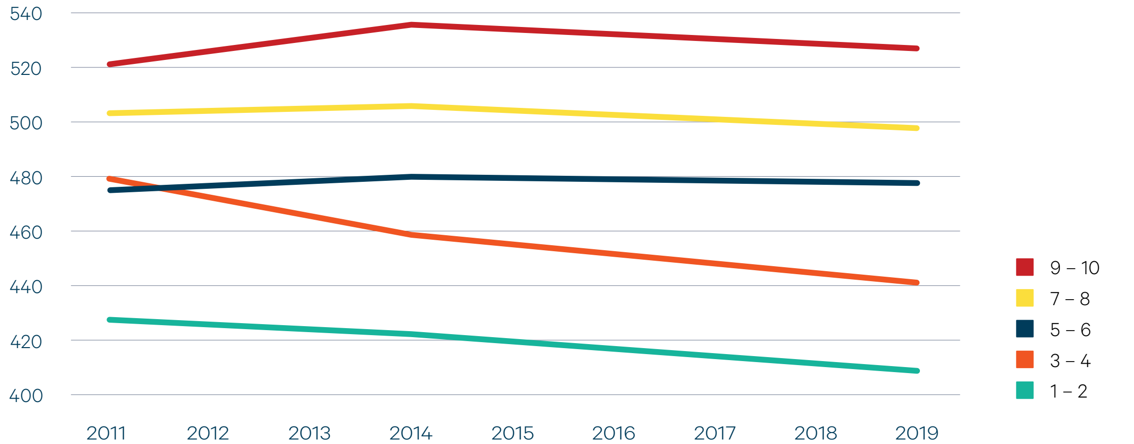
Year 9 maths – achievement by ethnicity



Source: Trends in International Mathematics and Science Study (TIMSS)

FIGURE 5

Year 9 maths – achievement by school decile



Source: Trends in International Mathematics and Science Study (TIMSS)

It is the Panel's view that for student achievement and learning to improve, change needs to occur at a systems level, at the classroom level and in the curriculum the students experience – **not just the official written curriculum documents**. Further change needs to occur at a systems level to support teachers in providing opportunities for children to learn, and to engage parents in supporting their children's learning. The Panel's deliberations have resulted in 14 recommendations that situate this information within a broadly conceived mathematics education system that the Panel believes needs adjustment at all levels in order to make a new or extant curriculum work in practice. The fact that many of the issues are closely interlinked means that the recommendations cannot be meaningfully addressed individually.

If all ākonga students in Aotearoa New Zealand knew and could perform all the achievement objectives in the current mathematics and statistics curriculum that are suggested for their year level, our country's achievement profile would look completely different. Therefore, the Panel's curriculum-related advice includes recommendations on what needs to accompany the implementation of a refreshed curriculum. The issues are fundamentally complex, arise from multiple sources, and require multi-part solutions across all levels of the system. There are no quick fixes; it will require a sustained and prioritised effort.

While cognisant of the importance of mathematics and statistics to the wellbeing and overall development of all ākonga students for successful and fulfilling lives, there is also a considerable socioeconomic opportunity cost, as well as other more direct costs, to Aotearoa New Zealand stemming from poor achievement in these disciplines and the growing disparities in achievement in them. Numeracy and higher-level mathematical skills are valued by employers and nations as never before due to their direct impact on productivity. For this reason, the Panel has also noted these socioeconomic impacts.

Finally, the Panel is clear that this report is not about improving Aotearoa New Zealand's scores on international tests; it is about achieving the best possible outcomes for our future citizens and the country and society they will live in. In line with other studies,[3] in both high-performing and low-performing nations, these data from international tests are simply reflections of the mathematical education of students, with the benefit of enabling mostly valid international comparisons.

[3] For example, *What to do about Canada's declining math scores*, Commentary 427, C.C. Howe Institute.

Hōkaitanga me Ngā Tikanga Whakahaere

Scope and Terms of Reference

Against this backdrop, the Panel was asked to comment on the following:

- The mathematical and statistical skills and knowledge needed for being a critically engaged citizen.
- The mathematical and statistical skills and knowledge learners need to know and by when, and the important cross-disciplinary links, taking into consideration the rapid changes and growth in computer science/ICT.
- The important 'big ideas' in mathematics and statistics that all learners need to develop through schooling.
- The relationship between numeracy and mathematics.
- Progress, assessment, and where the checkpoints in the mathematics education pathway should be.

The Ministry of Education intends to use the Panel's advice to inform the refresh of Mathematics and Statistics[4] in Te Marautanga o Aotearoa/The New Zealand Curriculum. While the terms of reference focus on mathematics and statistics content knowledge in various ways, evidence suggests that the positioning of content is not the main challenge, and there is already a good understanding of what must be taught and by when; rather, the system as currently configured does not allow many of our students to get there. Therefore, this report comments on these systemic issues as well as addressing the Terms of Reference where possible.

The Panel has primarily addressed mathematics and statistics learning in Years 1–10 and in English-medium schooling. The reason for focusing on Years 1–10 is that during this period the foundations are laid for later engagement and success in mathematics and statistics, and it is during this period that learners' trajectories start to deviate in ways that impact engagement with senior secondary mathematics and statistics. While Māori-medium expertise was represented on the panel, overall, the focus was on English-medium schooling. In addition, advice to the Panel made plain that more work was needed to understand the progress and engagement of ākonga students in Māori-medium settings. There is a clear need for an appropriate review, such as that of this panel, in the Māori-medium sector, where there are less data. Thus, the panel's first recommendation is:

RECOMMENDATION 1.
That a group of experts in Māori-medium mathematics is convened to discuss the current state of pāngarau education, collate evidence, and make independent recommendations to the Ministry of Education.

As noted, a catalyst for the convening of this Panel was New Zealand's below par performance in TIMSS 2018/19. While the Māori-medium sector did not participate in TIMSS and there are minimal

[4] See our later section on 'Big Ideas' to gain an understanding of how the words pāngarau and tauanga are understood.

recent quantitative data available on student performance for this cohort of students, there are data from earlier projects. For example, the Poutama Tau Project[5] and more recent qualitative student reports and anecdotal evidence suggest that there are several enduring issues confronting pāngarau education. These include opportunities for teacher professional learning,[6] initial teacher education,[7] and linguistic issues.[8]

The key to any response to address equity and student achievement issues in pāngarau education and the key goals of Māori-medium education (language and knowledge revitalisation), requires teachers, parents, and the community's aspirations for students' education to be prioritised. Historically, the approach taken to address pāngarau education issues highlighted the tensions between the distinctive needs and priorities of minority Indigenous language education (Māori-medium) developments and those of majority national (English-medium in Aotearoa New Zealand). These include the tendency of agencies to frame initiatives in terms of mainstream education in the first instance and to then assume that these will naturally 'translate' to the Indigenous language education context. Contrary to this, proponents of Māori-medium education have successfully argued that initiatives must be derived from their own educational goals, practices, and contexts, including, centrally, the commitment to Indigenous language and knowledge revitalisation via schooling. This group of experts in Māori-medium mathematics could ensure that:

1. an analysis is carried out of what research data are available to identify the issues impacting pāngarau education
2. there is then discussion of the current state of pāngarau education and make recommendations directly to the Ministry of Education.

The Panel expects there will be considerable international interest in such a report as other nations seek to consider similar issues in their own context.

[5] Trinick, T., & Stevenson, B. (2006). Te Poutama Tau 2006: Trends and patterns. *Findings from the New Zealand Numeracy Development Projects 2006*, 44–53. Wellington: Ministry of Education.

[6] Meany, T., Trinick, T, & Fairhall, U. (2013). One size does NOT fit all: Achieving equity in Māori mathematics classrooms. *Journal for Research in Mathematics Education* 44(1): 235-263

[7] Hōhepa, M., Hāwera, N., Tamatea, K., & Heaton, S. (2014). *Te puni rumaki: Strengthening the preparation, capability and retention of Māori medium teacher trainees* (Final Report). Wellington: Ministry of Education

[8] Trinick, T., & May, S. (2013). Developing a Māori language mathematics lexicon: Challenges for corpus planning in Indigenous language contexts. *Current Issues in Language Planning* 14(3-4): 457–473.

Ngā Āhuatanga Matua e Hoahoa ana i ngā Putanga Mātauranga Pāngarau, Tauanga hoki o Aotearoa i ngā Kura Auraki

Key Factors Shaping Aotearoa New Zealand Mathematics and Statistics Educational Outcomes in English-Medium Schools

This section presents three of the four themes that emerged from the Panel's consideration of English-medium mathematics and statistics education in Years 1–10. Equity, the fourth theme, is considered in a later section.

Slippage

The current achievement and progress profile for ākonga students shows that there is a wide gap between mathematics and statistics outcomes for low and high achievers in Aotearoa New Zealand.

Further, progress through our education system widens rather than narrows this gap (see Figure 3). While low achievement and slow progress is clearly associated with socioeconomic status, our education system at present exacerbates existing differences linked to wider societal trends rather than ameliorating them.

Slippage leads to pedagogical challenges: how to cater for wide (and widening) differences in the level of knowledge and skill in every class; how to set and maintain appropriately high expectations for ākonga student achievement and progress; and how and when to introduce more sophisticated concepts to ākonga students who seem 'stuck' in earlier material. Pedagogical responses to these pressures, such as class 'ability grouping' (addressed later), staying within a 'safe zone' for ākonga students by not moving ahead with new ideas, and an apparent underestimation of ākonga student potential for progress, seem to have become entrenched over time. This has been inadvertently contributing to the very problems teachers are trying to solve.

The New Zealand Curriculum's open design encourages local flexibility and provides space for schools to tailor curricula to their communities. In Mathematics and Statistics, this flexibility means there is no mandated national approach to be followed and little guidance for teachers. Broad, two-year levels allow the postponement of teaching important skills and ideas until 'later'. A fundamental contradiction is that schools and teachers are afforded flexibility in mathematics and statistics, but they are not all given the support and guidance to use this flexibility in ways that will improve outcomes and system equity. Instead of producing rigorous contextualised learning, the lack of central direction contributes to delays in introducing more difficult material and slows progress.

This year-on-year increase in the range of attainment in each class, and the associated lack of progress for many students, is what we refer to as 'slippage'. It is not just a problem in Aotearoa New Zealand. Those involved in the new Australian curriculum design note that the average Australian student has fallen behind by nearly 14 months of schooling since 2003 according to

PISA results.[9] The report on curriculum redesign specifically notes that "if students do not learn what they are meant to learn at a given time, they fall behind, which causes them to continue to fall behind".[10] Strategies adopted in the United Kingdom to address related issues have also been discussed.[11]

How content is taught also impacts progress. There are two elements to this: how learning experiences are organised and what the nature of those experiences is. In terms of how learning is organised, within-class grouping (especially 'ability grouping') requires preparing multiple lessons, which creates more work for teachers and significantly reduces the teacher contact time each student has. This results in low engagement levels and poor learning outcomes (see Recommendation 5). The TIMSS data (see also the later references[66, 67]) indicate that within-class grouping may offer no benefit (effect size 0.18) and PISA 2012 showed it to be negatively correlated to mathematics performance in every country. There will naturally be times when activities are best done in smaller groups, but the general teaching and learning of mathematics and statistics should not always be done in small groups within a class. In terms of the nature of learning experiences, research suggests that a range of approaches including problem solving, learning to reason and justify, structured practice, and memorisation can be effective. Knowing how and when to use a range of pedagogical approaches is necessary for making content accessible to learners.

Recommendations 2–7 address these concerns. The recommendations focus on providing all ākonga students with regular, focused, accessible opportunities to learn mathematics and statistics and teachers and schools with support to design high-expectations programmes that introduce ākonga students to skills and knowledge at a pace that supports progress in line with curriculum expectations.

Teacher knowledge of mathematics and statistics

Greater teacher content knowledge of mathematics and statistics for teaching positively impacts ākonga students learning.[12] While there are a number of ways to conceptualise teacher knowledge for teaching mathematics and statistics, the common link is that more is needed than just being able to do the mathematics and statistics yourself.[13] There is strong evidence that initiatives that increase teacher knowledge of mathematics and statistics *for teaching* are very likely to improve outcomes for learners. This includes understanding mathematics and statistics, knowing the ways that mathematics and statistics are commonly learned, what the common mistakes and pitfalls are for learners, and how best to represent and model the ideas for learners, as well as how to connect the mathematical and statistical ideas to learners' worlds and motivations.[14]

[9] Joint Statement AAMT, AAS, ATSIMA, MERGA, AMSI.

[10] Center for Curriculum Redesign. (April 2021). Mathematics for a modern world: standards for a mathematically literate society.

[11] https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics

[12] Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. *Journal of Mathematics Education* 2: 147–164; Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education* 59: 389–407; Hill, H.C., Rowan, B., & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal* 42(2): 371–406.

[13] Ball, Thames, & Phelps. (2008); Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). https://www.sciencedirect.com/science/article/pii/S0742051X1300053X. *Teaching and Teacher Education* 34: 12–25; Rowland, T. Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge. *Journal of Mathematics Teacher Education* 8: 255–281.

[14] Ball, Thames, & Phelps. (2008); Rowland, Huckstep, & Thwaites. (2005); Goldenberg, P., & Mason, J. (2008). Shedding light on and with example spaces. *Educational Studies in Mathematics* 69(2): 183–194.

Building and maintaining high levels of teacher content knowledge for teaching mathematics and statistics in Aotearoa New Zealand schools is a long-term challenge. NMSSA results show that a very small percentage of Year 4 and Year 8 teachers in Aotearoa New Zealand had specialist knowledge of mathematics – also a finding in TIMSS 2019. Nearly half of Year 4 teachers were only 'moderately confident' in teaching any strand of the mathematics and statistics curriculum; while Year 8 teachers expressed greater confidence, there was still a sizeable proportion who were only 'moderately confident'. Of Year 5 teachers, 91% felt 'very well prepared' to teach number concepts; this dropped to 75% of teachers for geometry, measurement, and statistics, and only 56% for algebra. Students with teachers who felt well prepared had a higher average TIMSS score than those with teachers who felt not well or somewhat prepared.

Knowledge of mathematics and statistics content in people who want to become teachers is highly variable, as could be expected from the achievement and progress statistics in TIMSS, PISA, and NMSSA data. Evidence from entry into initial teacher education suggests that a majority of primary teacher candidates cannot answer mathematics questions appropriate for curriculum Level 4.[15] More recent research[16] indicates a persistence of this problem, particularly in teaching 'mathematics for numeracy' in the New Zealand primary school classroom.

This lack of disciplinary knowledge poses a challenge for teacher preparation and ongoing professional learning. Knowing that the people who want to be teachers may not have strong mathematics and statistics content knowledge will influence teacher education provision. The magnitude of the difficulty, exacerbated by negative experiences and attitudes in some teacher candidates, means that career-long input may be needed to support teachers to provide sound mathematics and statistics education for ākonga students. There is research supporting positive outcomes when actively addressing mathematical competence in ITE teachers programmes.[17]

TIMSS 2019 results show that only 59% of Year 4 students and 61% of Year 8 students were taught by teachers who had received professional learning on mathematics content in the past two years. This percentage represented a significant decrease in access to professional learning since 2014 (NMSSA). About half the teachers surveyed by the NMSSA in 2018 felt their professional support for teaching mathematics was good or very good (52% at Year 4 and 49% at Year 8). This means that about half the teachers thought their professional support was fair, poor, or very poor.

Providing a specialist teacher of mathematics for all learners is very challenging, particularly in secondary schools, and especially for schools that are hard to staff. 'Out of field' or 'non specialist' teachers are often recruited from within schools to cover mathematics and statistics teaching. These teachers are more likely in need of additional professional learning in mathematics and statistics content knowledge for teaching.

[15] Young-Loveridge, J., Bicknell, B., & Mills, J.P. (2012). The mathematical content knowledge and attitudes of New Zealand pre-service primary teachers. *Mathematics Teacher Education and Development* 14(2): 28–49.

[16] Mills, J.P. (2018). Investigating the professional knowledge of New Zealand primary school teachers when teaching mathematics for numeracy (Thesis, Doctor of Philosophy (PhD)). University of Waikato, Hamilton, New Zealand. Retrieved from https://hdl.handle.net/10289/11696

[17] Ingram, N., Linsell, C., and Offen, B. (2018) Growing mathematics teachers: Pre-service primary teachers' relationships with mathematics, *Mathematics Teacher Education and Development*, 2, 41-60.

These issues are not new – they have been described in research up to two decades ago. For instance, in 2000 Davies and Savell[18] assessed the previous mathematics achievement and basic mathematical competency of student teachers enrolling in an Early Years programme of teacher education focused on teaching children aged zero to eight. Almost a third of the student teachers had not studied mathematics beyond the fourth form (equivalent Year 10). In another study[19] (including the original study of Paterson, Hurring and Barton) the implication drawn is that many student teachers (in their studies) did not enter into initial teacher education with the kinds of subject knowledge that would support effective teaching. It was noted in that study that the depth of content knowledge required to successfully engage secondary school students in mathematical learning equated to second-year university-level mathematics.

The discussion above has focused on mathematics as there are comprehensive data from NMSSA. The Panel points out that there is much (and often similar) supporting evidence for the need for continuous professional learning specifically for statistics teachers in the statistics education literature. The important resource document *Statistics Education of Teachers* canvasses much of it.[20]

Further, it must be noted that, for good reason, many teachers may simply not be aware of contemporary pedagogical research in fast-moving areas of any discipline, with statistics being a key example. New Zealand has internationally recognised statistics education researchers who have uncovered areas of weakness for teacher knowledge as well as strategies to improve their understanding. For instance, seemingly simple graphs have a rich conceptual repertoire underpinning them, yet research work in this area is not generally known by teachers or even initial teacher educators. Understanding the research and how it applies to their teaching would enhance their professional learning and pedagogy.

Recommendations 7–12 address the concerns of the Panel regarding teacher disciplinary and pedagogical knowledge. Together they suggest a programme of professional support and learning that focuses on consolidating and increasing teachers' mathematics and statistics knowledge for teaching and associated pedagogies, suggesting recognition for teachers who have or will develop specialist expertise, and the recruitment of more people with strong mathematics and statistics knowledge into teaching.

Leadership

The Panel sees the current state of mathematics and statistics education as the outcome of a system, not the product of individual failings. This system view has implications for national leadership of mathematics and statistics education, as well as for teacher education providers and schools.

Aotearoa New Zealand's education system is characterised by high levels of school autonomy. In recent years, this autonomy has extended to the procurement of professional learning, with the disestablishment of centralised professional learning provision. The Panel's views on this echo

[18] Davies, N., & Savell, J. (2000). *"Maths is like a bag of tomatoes": Student attitudes on entry to an early years teaching degree.* Teacher Education Forum of Aotearoa New Zealand conference.

[19] Cameron, M., and Baker, R. (2005). Research on initial teacher education in New Zealand: 1993–2004 literature review and annotated bibliography (29).

[20] https://www.amstat.org/asa/files/pdfs/EDU-SET.pdf

the Tomorrow's Schools report:[21] in some circumstances, strong, central leadership is needed to promote equity of outcomes and to raise system capability. The Panel believes that mathematics and statistics education is one such area. Consequently, access to professional learning and development (PLD) opportunities for teachers in mathematics and statistics education should not be contestable and should be easily accessible.

Remedying the problem of significant numbers of under-prepared teachers in mathematics and statistics classrooms will likely take considerable time. The delivery of suites of excellent supporting material from a central authority, written by content-matter and pedagogical experts, is critical to improving the situation in the short-to-medium term. This will also relieve the burden of decision on teachers who are expected to act as 'curriculum designers' and have to regularly choose from a vast array of possible materials to work with. Models for professional development and classroom support that work in schools where well-qualified teachers are easy to attract may not be fit for purpose in hard-to-staff schools. Likewise, in fast-moving disciplines resources date quickly and school textbook writers struggle to keep up. Centralised support from the Ministry of Education by knowledgeable staff to facilitate these sorts of decisions and support schools in choosing materials or professional development options seems an obvious way to move forward.

Recommendations 11 and 12 directly address this theme, and recommendations 2–9 seek strong, sustained national leadership from the Ministry of Education for the curriculum, support for schools and teachers, and professional learning.

[21] Tomorrow's Schools Independent Taskforce. (2018). Our schooling futures: Stronger together Whiria Ngā Kura Tūātinitini. Wellington: Ministry of Education.

Te Pāngarau, Tauanga me te Porihanga

Mathematics, Statistics, and Society

What mathematics and statistics do critically engaged citizens need to know?

Critical engagement refers to the ability of people to ask questions of the world around them, to investigate and be inquisitorial, and reach reasoned opinions. Critically engaged citizens are able to both participate in society and reason about the wider implications of societal actions. These skills facilitate the good function and development of society through encouraging social justice, good policy and governance, economic efficiency, and social capital.

The need for people to be numerate – that is, to be able to calculate, estimate, and measure – are basic outcomes required of any mathematical education. In this technological age, the need for innovation, problem-solving, and decision-making skills are desirable outcomes for all ākonga students. Mathematics and statistics education provides an opportunity for them to develop these skills, helping them to become innovative and flexible problem solvers. Critically engaged citizens will naturally appreciate the connections between mathematics and life, and will exploit these connections. Mathematics and statistics provide ways for them to learn about and critique the world, and the reasoning and critical thinking skills they develop will support their day-to-day choices.[22]

For example, a critically engaged citizen is someone who:

- understands orders of magnitude and real-world numbers in context as well as understanding the consequences of exponential growth and decay
- has a basic understanding of risks and their probabilities – for instance, a citizen should understand how likely it is that smoking or not being vaccinated will significantly impact their health (10–15% of smokers develop lung cancer) or how likely it is to win lotto (buying a ticket a week, it will take you on average about 7,000 years to win)
- understands how uncertainty is always present in future projections, for example, in areas such as personal financial planning, predicting usage of a new transport structure, and the future water quality from a change in land-use practices
- has basic financial literacy and can manage a budget, in that, not only are they able to manage a household budget, but can manage more complex activities like scheduling payments and saving. Financial literacy includes understanding interest and debt, such as the concepts that borrowed money must be repaid eventually, generally with interest, and that interest accumulates such that the longer the repayment period, the more you will pay on top of the original amount

[22] Gravemeijer, K., Stephan, M., Julie, C., Lin, F-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? 15: 105–123. https://link.springer.com/article/10.1007/s10763-017-9814-6

- is a critical consumer of data.[23] They understand that data are needed to learn more about them and their world, and that evidence from good data outweighs that from anecdote. They ask themselves questions such as where did the data come from, why were the data collected, and what biases in the way the data were collected are present? Paying attention to these questions helps form and communicate reasoned opinion and habituates critical thinking
- appreciates that issues of data ethics, privacy, confidentiality, and ownership are paramount considerations for a functioning democracy as discussed by Te Mana Raraunga, Māori Data Sovereignty Network[24]
- understands the difference between correlation and causation, what are appropriate conclusions to infer and what statistically significant means in various contexts
- understands and is able to reason from graphs, data visualisations, and so on, including using their pertinent features, and more generally understands the presentation of data and drawing inferences from them.

Equity and social justice

Competencies in mathematics are so important in education, work, and life that the subject can be described as a gatekeeper. A high-quality mathematics and statistics education system that ensures equity for all learners is one in which achievement results are not influenced by ethnicity or socioeconomic status. Currently, the New Zealand education system produces inequitable outcomes. There are large differences in achievement in relation to socioeconomic status (NMSSA), and Māori and Pāsifika students are over-represented among lower achievers and under-represented among higher achievers. There is also a strong correlation between ethnicity and socioeconomic status. A key change needed for mathematics and statistics education is effective delivery to all learners, including Māori and Pāsifika students and those of lower socio-economic status.

For over 120 years, there has been a range of overt policies that excluded Māori identity from the classroom including banning the use of Māori language and cultural knowledge. With a history of the roles of schools in assimilation/integration of European beliefs, attitudes, and practices,[25] Māori student underachievement, in the main, was put down to parental apathy and indifference.[26]

This belief has lingered to this day and now is also used to explain Pāsifika student underachievement. Frequently, the cultural background of both Māori and Pāsifika students has been perceived as a deficit within the schooling system.[27] This study found that teacher expectations in New Zealand secondary classrooms were lowest for Māori and Pāsifika students.

[23] Gould, R., Wild, C.J., Baglin, J., McNamara, A., Ridgway, J., and McConway, K. (2018). Revolutions in teaching and learning statistics: A collection of reflections (457–472). In D. Ben-Zvi, K. Makar, & J. Garfield (Eds), *International handbook of research in statistics education*. Cham, Switzerland: Springer.

[24] https://www.temanararaunga.maori.nz

[25] Smith, J.P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education* 27(4): 387–402. pubs.nctm.org

[26] Bishop, R., & Glynn, T (1999). Researching in Māori contexts: An interpretation of participatory consciousness. *Journal of Intercultural Studies* 20(2): 167–182. DOI: 10.1080/07256868.1999.9963478

[27] Hunter, R., & Hunter, J. (2018). Maintaining a cultural identity while constructing a mathematical disposition as a Pāsifika learner. In E. A. McKinley & L. Tuhiwai Smith (Eds.), *Handbook of Indigenous education*. Singapore: Springer.

Teachers referred to perceived deficits in home backgrounds and attitudes of this group of students by stating that Māori and Pāsifika students lacked motivation, goals, aspirations, and parental support. Decades earlier, studies by Jones[28] found the same perceptions. This issue simply must be dealt with.

To address the ongoing inequity in the current education system, forms of teaching that provide opportunities embedded within culturally sustaining[29] and asset-based teaching and assessment that recognise and privilege Indigenous knowledge systems[30] are needed. This includes embedding into mathematics and statistics classrooms various types of pedagogy that align with both students' identity and values and in developing tasks that connect faithfully with the cultural and social contexts of students' daily lives.[31 32 33 34] A fuller discussion of these issues can be found in Bishop et al.[35]

Education can be a force for positive social change and emancipation, but in Aotearoa New Zealand, the mathematical education system appears to be replicating and exacerbating social inequity, not reducing it. For instance, students from decile 7–10 schools are two or three times more likely to gain University Entrance qualifications than those from deciles 1–3. This achievement gap between higher and lower socioeconomic groupings is particularly large in Aotearoa New Zealand when compared to international benchmarks.[36] Further, the TIMSS data show a decline in performance in Mathematics and Statistics on average as deprivation levels increase. This is not unexpected. However, performance declines more steeply in New Zealand than in other countries. This is perhaps one of the strongest pointers to failure of our system. For instance, New Zealand's 'average home resources' are among the best in the survey at TIMSS Grade 4 and Grade 8. Those with 'many resources', 17% in New Zealand, score 540 (international average 546); 'some resources', 77% in New Zealand, score 475 (488); and 'few resources', 6% in New Zealand, score 414 (433), showing a more precipitous fall than most other nations.

[28] Jones, A. (1989). The cultural production of classroom practice. *British Journal of Sociology of Education* 10(1): 19–31. DOI: 10.1080/0142569890100102

[29] Paris, D. (2012). Culturally sustaining pedagogy: A needed change in stance, terminology, and practice. *Educational Researcher* 41(3): 93–97.

[30] Koya Vaka'uta, C.F. (2017). Rethinking research as relational space in the Pacific: Pedagogy and praxis. In U L. Vaai, & A. Casimira (Eds.). *Relational hermeneutics: Decolonisation and the Pacific Itulagi* (65–84). Suva, Fiji: Pacific Theological College, University of the South Pacific.

[31] Hunter, J., & Miller, J. (2020). Using a culturally responsive approach to encourage early algebraic reasoning with diverse young learners. *International Journal of Science and Mathematics Education* (online first).

[32] McKinley E., Stewart J. & Richards, P. (2004). Māori students in science and mathematics. Set 3: 9–13.

[33] Trinick, T., & Meaney, T. (2017). Indigenous teacher education: When cultural enquiry meets statistical enquiry. In A. Downton, S. Livy & J. Hall (Eds.), *40 years on: We are still learning. Proceedings of the 40th annual conference of the mathematics education research group of Australasia* (514–521). Melbourne: MERGA.

[34] Trinick, T., Meaney, T. & Fairhall, U. (2017). Cultural and mathematical symmetry in Māori meeting houses (wharenui). In M. Rosa, L. Shirley, M. E. Gavarrete, & W. V. Alangui (Eds.), *Ethnomathematics and its diverse approaches for mathematics education* (235–255). New York: Springer.

[35] Bishop, R., Berryman, M., & Wearmouth, J. (2014). Te Kotahitanga: *Towards effective education reform for Indigenous and other minoritised students.* Wellington: NZCER Press. https://www.nzcer.org.nz/nzcerpress/te-kotahitanga

[36] Affluence level of households, according to Principal reports in TIMMS, is similar to the international average. At Grade 4, only the 'More affluent' category beat the international average (526 vs 521), with steeper than average declines to 'More disadvantaged' (441 vs 479) [Ex 6.2, p. 312]. Similar pattern at Gr 8 [Ex 6.4, p. 314].

Similarly, the number of students feeling tired or hungry is similar to international averages, and, while we wish this for none of our ākonga students, the drop in performance in schools with more hunger is steeper in New Zealand than international averages. Further, New Zealand has a higher incidence of 'students not ready for instruction' at Grade 4 and again the drop-off in performance is steeper in New Zealand than international averages. In fact, this gap accelerates between Grade 4 and Grade 8. By contrast, a number of other OECD countries manage to combine high educational achievement with smaller gaps in achievement between students and between schools.

Recommendations 2, 3, 8, 10, and 14 directly address these concerns. Collectively, these recommendations would provide ākonga students with mathematics and statistics teaching that valued their culturally identity while focusing on high expectations for all. They would also address the support that teachers need to develop culturally sustaining pedagogy and recognise the strengths of their students.

Economic and social consequences of low levels of mathematical skills for society and individuals

This section highlights some of the evidence on the socioeconomic consequences of low levels of mathematical skill in our communities. The commentary, data and analysis below support the case for the Panel's recommendations to be taken seriously. The Panel would like to see more public discussion on the individual- and society-level economic consequences of mathematical attainment levels, given data appear to show that the impacts are the harshest for those groups over-represented in the lowest attainment levels. The long-term study[37] paints a sobering picture of these impacts.

Improving individuals' outcomes is a key goal of education. Aotearoa New Zealand must meet the growing demand for people equipped with higher-level, economically valuable skills if it is to reshape its economy around high-value, knowledge-intensive activities within an increasingly competitive global economy. In particular, this means ensuring that businesses have access to science, technology, engineering, and mathematics skills as these are central to developing innovative products and services that can be effectively positioned in world markets.

While the consequences of poor numeracy and low levels of mathematical and statistical skills are often discussed, there is a surprising lack of studies identifying and quantifying this impact at both the macro-societal level and micro-individual level. Numeracy is identified and measured along with literacy as a key component of human capital, and so the arguments for human capital and mathematical skills could be viewed as analogous. Indeed, one of the suggested problems with identifying the impact of poor mathematics attainment is that it is so strongly correlated with overall education performance, in particular literacy. Simply put, poor mathematics attainment is symptomatic of a struggling education system and poor education outcomes overall. Poor mathematics attainment has been shown to impact national economies significantly and negatively.

[37] The expression, experience, and transcendence of low skills in Aotearoa New Zealand. https://workresearch.aut.ac.nz/research/low-literacy-and-numeracy-research

[38] https://www.oecd.org/skills/piaac/

The studies that do exist paint a worrying picture of these impacts. Within the OECD, the Programme for the International Assessment of Adult Competencies[38] (PIAAC) is conducted in over 40 countries through surveying and measuring adults' proficiency in key information-processing skills: literacy, numeracy, and problem solving. It also gathers information on how adults use their skills at home, at work, and in the wider community. The numeracy framework[39] and its later refinements measure both literacy and numeracy skills on a 500-point proficiency scale, broken down into five proficiency levels: New Zealand numeracy results show nearly a fifth are at or below Level 1 (lowest) and the majority are at Levels 2 or 3.

The OECD figures show that in New Zealand there is about a 9% premium in wages between each numeracy level (independent of whether these skills are directly related to the type of work). The long-term study[36] notes that in practical terms, this means that more than one in five New Zealand adults finds everyday tasks such as reading and understanding job advertisements challenging. It goes on to assess the impact of low numeracy and literacy on a number of societal concerns such as health outcomes and involvement with the criminal justice system.

While the societal impacts of low numeracy are clear, we have found no New Zealand data or modelling that provide an estimate of the economic impact. Looking abroad, an Australian study estimates up to 40% of the association between education and employment is attributable to literacy and numeracy skills.[40] Additional gains in numeracy lead directly to gains in employability, productivity, and a host of other beneficial outcomes for individuals, communities, and their country. Higher literacy and numeracy skills are associated with better labour market outcomes (employment and wages) and Australian econometric modelling shows that:

- an increase in literacy and numeracy by one skill level[41] is associated with an increased likelihood of employment of 2.4 and 4.3 percentage points for men and women, respectively, regardless of whether a person had a degree, diploma/certificate, or Year 12 education
- an increase in literacy and numeracy by one skill level is associated with about a 10% increase in wages for both men and women. This positive association is equivalent to that of increasing educational attainment from Year 11 to Year 12 or to a diploma/certificate
- more than half of the 'penalty' that affects the wages of people with a non-English speaking background is explained by their lower literacy and numeracy skills.

Further afield, a United Kingdom[42] study shows that the productivity of jobs requiring mathematical ability (Levels 4 and 5) is more than twice that of the national average. That Deloitte study comprehensively identifies the significant value of mathematics to the economy of the United Kingdom. However, roughly four in every five adults there struggle with mathematical literacy[43]

[39] PIAAC Numeracy Expert Group (2009). "PIAAC Numeracy: A Conceptual Framework", *OECD Education Working Papers*, No. 35, OECD Publishing. http://dx.doi.org/10.1787/220337421165

[40] https://www.pc.gov.au/research/supporting/literacy-numeracy-skills/literacy-numeracy-skills.pdf

[41] A 'skill level' is defined as a band on standardised literacy and numeracy testing (PIAAC). There are five skill levels identified, though few attain skill Levels 4 and 5. More details can be found at: http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4228.02011-12

[42] Measuring the Economic Benefits of Mathematical Science Research in the UK.

[43] https://www.nationalnumeracy.org.uk/about/what-issue

and 49% of working age adults have the numeracy skills of a primary school child. Studies have linked this to likely deep skills shortages in the near future.[44] Poor numeracy was also shown to negatively impact the health and wellbeing of citizens, and was estimated to cost that economy around 1.2% of GDP.[45]

Supporting these arguments, early research by Bynner and Parsons[46] suggested that adults with poor numeracy skills could be characterised by particular employment trajectories:

- leaving full-time education early and with no qualifications
- patchy employment with periods of casual employment and unemployment
- jobs that were low skilled, poorly paid, and offered few chances of training and promotion.

Each of these trajectories carries negative economic impacts for an individual. There is also evidence that numeracy carries a larger penalty in the labour market than literacy in both Australia and the United Kingdom (and the penalties are somewhat harsher for women).

The report[47] for the United Kingdom makes it plain that in the workforce there is a steady shift away from manual and low-skill jobs towards those requiring higher levels of management expertise and problem-solving skills, many of which are mathematical in nature, and that "employers emphasised the importance of people having studied mathematics at a higher level than they will actually use. That provides them with the confidence and versatility to use mathematics in the many unfamiliar situations that occur at work".

These international examples provide sobering implications for Aotearoa New Zealand's unfavourable performance in almost all international mathematical education benchmarks. Were the GDP loss predicted by the United Kingdom applied to the New Zealand context, the impact would be over $4 billion per annum. The poor labour market outcomes noted by both the United Kingdom and Australian studies further suggest that some 20–40% of New Zealand children will be disadvantaged in employment or income statistics as a result of numeracy attainment below their potential. Further, the Australian study suggests that economic participation, mobility, entrepreneurship, and innovation will be stifled, while the United Kingdom study highlights the resulting social, health, and justice costs faced by society.

This analysis suggests low levels of mathematical skills and attainment are and will remain significant impediments to Aotearoa New Zealand's future wealth and wellbeing.

[44] https://industrialstrategycouncil.org/sites/default/files/UK%20Skills%20Mismatch%202030%20-%20Research%20Paper.pdf

[45] https://www.probonoeconomics.com/national-numeracy-calculating-the-cost-of-low-adult-numeracy

[46] Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?* London: National Research and Development Centre for Adult Literacy and Numeracy. https://core.ac.uk/download/pdf/111651.pdf

[47] Advisory Committee on Mathematics Education. (2011). *Mathematical needs: Mathematics in the workplace and in Higher Education.* London: The Royal Society.

Te Hononga o te Reo Pāngarau ki te Pāngarau me te Tauanga

The Relationship Between Numeracy and Mathematics and Statistics

The PIAAC numeracy framework exhaustively works through the many potential definitions of numeracy as does the subsequent review of that framework, which recommends a range of improvements and enhancements, including the definition and elaborations of adult numeracy used, and the assessment content. Many suggestions from the PIAAC report arise out of the concern that the existing framework and assessment do not fully reflect the realities of the skills and knowledge adults now need to succeed in work, life, and citizenship in the 21st century.[48]

The PISA 2021 Mathematics Framework defines mathematical literacy (an alternative term for numeracy) as follows:[49]

> Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens.

In Aotearoa New Zealand's context, numeracy is often associated with preparing children for using mathematics in adulthood beyond school, and is often used as an exit qualification for school leavers. This has been evidenced recently in the new proposed NCEA Numeracy standard. The opening statements of the 2020 consultation documents on numeracy[50] refer to the satisfaction that learners experience by understanding situations using mathematics and statistics, the joy in using mathematical/statistical ideas to improve the lives of others, and the experience of mathematics statistics as a creative and empowering human endeavour. The Panel endorses this aspirational view:

> Foundational numeracy refers to the knowledge and capabilities that enable learners to access further learning, develop important life skills, engage in employment and in their communities. In Aotearoa New Zealand, this includes an understanding of how to participate in a bicultural society. Numerate people can interweave mathematical/statistical processes and content. This interweaving allows them to manage the mathematical and statistical demands of a range of situations, and supports them to find solutions that service others, promote innovation, empower communities, and care for our planet.

[48] Tout, D., Coben, D., Geiger, V., Ginsburg, L., & Hoogland, K. (2017). *Review of the PIAAC Numeracy Assessment Framework: Final Report 2017.* Australian Council for Educational Research (ACER).

[49] https://www.oecd.org/pisa/pisaproducts/PISA2021_Mathematics_StrategicDirectionPaper.pdf pp. 7

[50] Unpacking Numeracy: https://consultation.education.govt.nz/ncea/ncea-literacy-and-numeracy-questionnaire/supporting_documents/Numeracy Learning Matrix and Unpacking Numeracy.pdf

The knowledge, understanding, processes, and application of mathematics and statistics that are explicit in most operational definitions of numeracy (including those above) are also the foundations of more advanced mathematics and statistics. The strong emphasis on practical elements in definitions of numeracy implies that ākonga students should have the capability and disposition to apply mathematical and statistical thinking and tools to their everyday lives. These abilities to identify, adapt, and learn from relevant mathematics are equally essential for the 'doing of mathematics and statistics' with a high-level discipline focus. Therefore, numeracy is both a goal in itself, and a step on the pathway to acquiring the mathematics and statistics needed for being:

- a critically engaged citizen, or
- an effective worker in the economy of the future, or
- a scientist at the forefront of research and development.

For these reasons, the Panel strongly cautions against drawing a distinction between numeracy and mathematics. Many mathematicians and statisticians would find their day-to-day activities encapsulated in the above definitions, and there seems to be little to be gained by creating two separate 'subjects'.

Ngā Ariā Nui o te Pāngarau me te Tauanga

Big Ideas of Mathematics and Statistics

There already exist many comprehensive lists of 'Big Ideas',[51] including some of those used by the Ministry of Education. Most of these lists focus on *content*: the big mathematical and statistical ideas that frame and underpin the school curriculum. Rather than offering another detailed dissection of content, the Panel has drawn on its own expertise as practitioners of the disciplines to emphasise broader themes.

Before 2007, the national curriculum documents listed statistics as a topic within the learning area of Mathematics. Since the last curriculum review, however, the New Zealand Curriculum has titled the learning area 'Mathematics and Statistics'. This move is a world first, and recognises statistics as a discipline in its own right within the primary and secondary education systems, noting that it has both distinct and complementary practices to mathematics.

In the early 1990s, experts in mathematics education and te reo Māori developed a standardised Māori mathematics vocabulary for the complete school curriculum.[52] Among the new terms were *pāngarau*, denoting mathematics, and tauanga, denoting statistics. *Pāngarau*, meaning 'multiple relationships', was chosen for mathematics rather than terms that emphasised number or calculation alone. This etymological decision recognises that mathematics is not merely arithmetic,

[51] Charles, R. I., & Carmel, C. A. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education* 7(3): 9–24; Clarke, D.M., Clarke, D., & Sullivan, P. (2012). Important ideas in mathematics: What are they and where do you get them? *Australian Primary Mathematics Classroom* 17(3): 13–18; Askew, M. (2013). Big ideas in primary mathematics: Issues and directions. *Perspectives in Education* 31(3): 5–18.

[52] Trinick, A. (2015). *Te Reo Tātai: The development of a mathematics register for Māori-medium schooling* (Doctoral dissertation). University of Waikato, Hamilton, New Zealand; Barton, B., Fairhall, U., & Trinick, T. (1998). Tikanga Reo Tātai: Issues in the development of a Māori mathematics register. *For the Learning of Mathematics* 18(1): 3–9.

but is concerned with multiple (*rau*) relationships (*pānga*) involving quantity, space, change, and abstract structures. The emphasis on relationships echoes a contemporary philosophical view of mathematics as "the study of structure",[53] in which structures (systems of relationships) are the focus, rather than objects in isolation. For example, number structures are not simply a collection of individual objects such as '5', '6', and '7', but a system of relationships in which each object derives its meaning from its relationship to other objects in the structure (such as, in the natural numbers, '6' is the successor to '5', and the predecessor of '7').[54]

Similarly, the te reo Māori word *tau*, meaning number and enumeration, forms the root of the term *tauanga*. Whereas number and enumeration were not emphasised in the choice of word for mathematics (number being only one of many relationships in mathematics), these concepts were embraced for statistics, as they nod to the idea of data, which are at the heart of the discipline.

As with other countries, in Aotearoa New Zealand 'Big Ideas' have been chosen to wrap around the curriculum. Big Ideas were seen as a potential vehicle for making mathematics education more integrated.[55] The ideas we articulate below are in this spirit and have been chosen to:

- include fundamental notions for developing confidence and capability in Mathematics and Statistics for individual learners
- connect our aspirations for critically engaged citizens with curriculum
- prepare students for technological change
- allow for a comprehensive basis for tertiary level study in STEM subjects.

The big ideas are not a list of things that must be learnt through schooling, but are beacons to guide detailed curriculum development. We suggest curriculum experts and teachers can 'work backwards' from the big ideas, making choices about age-appropriate content and learning activities that staircase learners to understand, appreciate, and apply these ideas as they progress.

The curriculum and learning activities that give effect to the big ideas should be enriched with authentic connections to other learning areas and questions of 'where, why and how did people use these ideas in the past?' and 'where are these ideas in use today?' In this way, learning can reflect the competitive edge mathematics and statistics gives to industrial and scientific advances; the influence of the disciplines in all models, tools, equipment, techniques, and algorithms; and that for millennia humans from all cultures have created and used mathematical tools to understand and change the world.

The big ideas reflect the need to respond to the contemporary reality that statistics as a discipline is changing rapidly – likewise, the way that mathematics and statistics are being used. While much of the current curriculum content in the mathematics threads is appropriate and does not need to change, there are some themes that are under-represented and connections between some strands are not explicit enough.

[53] Shapiro, S. (1997). *Philosophy of mathematics: Structure and ontology*, Oxford: Oxford University Press.

[54] Benacerraf, P. (1965 [1983]). What numbers could not be. *Philosophical Review* 74: 47–73. Reprinted in P. Benacerraf and H. Putnam (Eds.), (1983), *Philosophy of mathematics: Selected readings*, second edition. Cambridge: Cambridge University Press, pp. 272–294.

[55] https://cdn.qct.edu.au/pdf/Research%20Periodicals/QCTResearchDigest2015_11.pdf

1. Communication and visualisation are at the centre of the many ways that mathematics and statistics provides for framing, analysing, and solving real-world problems.

Mathematics and statistics provide multiple perspectives for looking at situations. For example, a daily activity like choosing how to travel from home to school has spatial features and time constraints, as well as sources of risk and uncertainty. Elements of this situation can be described numerically, graphically, statistically, and logically. Statistical and probabilistic thinking and knowledge help people learn more about their decision making. The increasing availability of powerful computer systems provides unprecedented opportunities for engaging with rich and accessible data sets, empowering people to think and form knowledge in new ways and earlier in their mathematical development. Other technologies[56] are also used as tools. Mathematics and statistics are creative, and their thinking styles and learning are interwoven with the way the subject is communicated and visualised. These are at the centre of mathematics and statistics, and are fundamental to deriving joy, value, and, ultimately, utility from these activities. Mathematical and statistical knowledge can be communicated orally, visually through numbers and symbols (tables, graphs, images, videos, writing), and through gesture.

2. Competence with numbers, arithmetic, and algebra underpins further learning in mathematics and statistics and builds confidence through use.

The existing New Zealand Curriculum strands on number, measurement, and algebra are essential because number systems give precise description of quantities and allow calculations to be performed accurately and efficiently. Arithmetic with numbers with one or two decimal places is useful in daily life. Mastery of calculation techniques is deeply connected with an understanding of number systems and is foundational to both further learning in mathematics and statistics[57] and practical skills.[58] For example, estimation approximates a numerical calculation by replacing some numbers with other numbers that are close and easy to compute mentally, giving the ability to rapidly check calculations to orders of magnitude or significant digits. Algebra is essential to fluency in the languages of mathematics and statistics. Confidence and competence with handling algebraic expressions is required when using abstract descriptions of mathematical and statistical situations.

3. Statistics is the science of learning from data.

Statistics has been defined as the science of learning from data, and of measuring, controlling, and communicating uncertainty.[59] We use statistics to learn about the world and the phenomena we see in it. Data are units of information that are created and collected through observation. Data collection must be designed. Data are recorded in a systematic way, and are used to provide evidence for predictions, decisions, and evaluating risk.

[56] Such as calculators, graphing apps, computer algebra systems, and so on.

[57] Basic facts and procedures for operations with rational numbers use notions of equivalence to transform calculations into simpler ones; rules of arithmetic and algebra are used together with notions of equivalence to transform equations and inequalities so solutions can be found; work with percentages, decimals, and fractions culminates in ability to manage proportional reasoning, which is critical to robust probabilistic thinking.

[58] Converting medicine concentrations of milligrams per millilitre into a liquid dosage based on milligrams per kilogram of bodyweight requires both confidence and competence with arithmetic and measurement; number supports an understanding of scale (how big is one hundred compared to ten thousand compared to one million? What is the difference between a medicine dose in milligrams versus grams?).

[59] Wild C.J., Utts J.M., Horton N.J. (2018) What Is Statistics? In Ben-Zvi, Makar, & Garfield J. (eds).

Data can be collected from many sources, and increasingly come from large computer-based systems. Not only is the recent growth in data availability unprecedented, new sources and types of data continually emerge. As we move forward into the data-centric world, statistics is shifting beyond traditional inference to computationally intensive approaches and new ways of processing and reasoning with it. For instance, data underpin machine learning, which then supports Artificial Intelligence decision-making processes.

4. Variation and uncertainty are described and explained via probability and randomness.

Variation is a natural feature of data collected from the real world. Variation, and the associated uncertainty from this, is at the heart of statistics. Recognising that variation is present in the real world means it needs to be incorporated in the statistical design phase and in the way the data are collected.

Quantifying variation requires an understanding of averages, randomness, and probability.[60] Distributions are the lens through which variation in data is viewed and understood. Visualising data in graphs and other formats enables variation, central tendencies, patterns, and exceptions (outliers) to be observed.

The omnipresence of variation implies a need for probabilistic reasoning, in which proportional reasoning with percentages and decimals is inherent. For example, what is the probability that I have a disease given I have a positive test? What is the relative risk, absolute risk, or factors increasing risk in a given situation? What is the meaning of a one-in-one-hundred-year event? Probabilistic thinking differs from mathematical reasoning in that the later seeks a deterministic solution, whereas statistical reasoning is not always about seeking a final answer.

5. Logic and reasoning are fundamental in mathematics and statistics.

Mathematical and statistical thinking includes proposing, communicating, dissecting, and rejecting ideas, and is part of the process of forming mathematical and statistical knowledge. Questioning and arguing are key activities, and logical reasoning provides the link between what we know about a situation and what we can deduce. Inductive reasoning is particularly important in statistics. Logic governs the story that is told from data and leads to questions about context and the way the data were gathered and analysed, to answer a statistical investigative question.

Statistical and probabilistic thinking are not intuitive and have to be learned to avoid jumping to conclusions too quickly. In mathematics, making mistakes and using reasoning to fix them are fundamental parts of the process of learning and creating new knowledge; the 'right answer' is seldom the whole thing. Mathematicians ask: Do I know this is true? Is it true in more general situations? How do I know? Logic governs the rules of mathematical proof; with proof, mathematical knowledge becomes certain.

[60] Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. (2020). Pre-K-12 *Guidelines for Assessment and Instruction in Statistics Education (GAISE) report II*. American Statistical Association and National Council of Teachers of Mathematics.

6. Abstraction makes mathematical and statistical knowledge into powerful and flexible tools.

Abstraction can make it clear that situations that look different are in some sense the same; thus, properties of one situation apply to others. Abstraction is the process of moving from a known situation to the mathematical one and delivers transferability. Abstraction may be curiosity driven or purposeful (to simplify a situation by identifying its essential features and make it easier to work with, or to generalise and increase its applicability). Structuring and thinking about data necessarily involves abstractions.

Examples include: moving consistently from addition of the numbers 0 and 1 leads to addition of all natural numbers – this embodies and creates the theory and relationships of the natural numbers; properties of abstract triangles underpin the surveying of large areas of land; patterns in the natural numbers lead to the notion of prime numbers, whose properties have unanticipated uses in securing digital communications; abstraction enables mathematicians and scientists to develop equations to determine how complex systems (such as the weather) evolve; the abstract notion of Brownian motion – first observed under a microscope – now underpins the pricing of complex financial products.

Abstract objects and their properties are named (variables) and encoded in notation and syntax. For example, equations can be formed where variables represent unknown (but sought) quantities and manipulated as if they are numbers. Productive abstract thinking in mathematics and statistics involves moving between the abstract and concrete world. The language of mathematics and statistics is specific and must be learned for users to read, write, and converse in these disciplines; and the language varies from informal to formal. Some abstractions should be learned by all students because they underpin understanding and use of mathematics and statistics in general.[61]

7. Statistical inquiry is an interrogative cycle in which data-derived knowledge is created and critiqued.

Questions in statistics come from the real world because of the fundamental human need to learn about how our world operates. Data are obtained and interpreted, and statistical thinking[62] can be used to unlock stories in the data that answer questions about the real world. Statistical inquiry is the process of posing a question, gathering, and analysing appropriate data, and critiquing and communicating the findings as evidence to the resolution of the original problem. In this way, real-world situations can be described and understood, and future outcomes can be predicted with various degrees of certainty. This inquiry process is an interrogative cycle, a conversation with the data, involving moving between the statistical and contextual world. Conclusions from statistical inquiry should be expressed carefully, using precise language that acknowledges sources and levels of uncertainty.

[61] For example, the natural, rational, and decimal number systems are useful and consistent extensions of the numbers {0, 1, 2, ... 9} where the rules of arithmetic hold; straight lines and common geometric figures have properties that can be used to model real-world situations; some objects and events can be classified according to their properties; a special rule (function) assigns each member of one set to a unique member of the other set; conditional probabilities provide a way of describing and calculating the chance of events occurring; inferences can be made from a sample of an unknown population.

[62] Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review* 67(3): 223–265. https://doi.org/10.1111/j.1751-5823.1999.tb00442.x

8. Modelling is used in mathematics and statistics to explain, predict, and understand the world.

Mathematics and statistics are interconnected with social, economic, and environmental concerns because models are developed to understand real-world phenomena, and can provide evidence that influences policy decisions. Assumptions and abstraction are used to create models that describe, explain, and predict real-world phenomena and data from almost all knowledge areas, and support decision making and the evaluation of risks.

There are many types of and purposes for modelling, and the extent of pre-existing knowledge about relevant quantities, relationships, and mechanisms varies widely. Similar variation occurs in how randomness is built in, the types of data used (real or simulated), and the way in which data are used within the model and for validation. As abstractions, models are necessarily distortions of reality, and conclusions from mathematical and statistical modelling should be expressed carefully, including explanations of the assumptions and limitations of the modelling approach.

Models are useful when the system of interest cannot be measured directly, or it is too dangerous, expensive, or unethical to do so. The modelling cycle includes making assumptions about the real (or artificial) situation, simplifying and structuring the situation into a model, running the model with suitable data, validating the solution against data, questioning the validity and sensitivity of the model assumptions, and interpreting the results back into the real situation for decision making. This process makes models robust, enabling them to be iteratively improved over time as more data become available, and as the purpose and use of the model results evolve. Models are evaluated in terms of the purpose for which they are created.

State-of-the-art mathematical and statistical modelling can be sophisticated and difficult. However, all learners should gain some experience with using and interpreting models. The detail must vary with level of schooling, but essential learning includes modelling situations with linear growth, those with exponential growth, and simple bivariate relationships with variation (for example, height and age). In higher levels of schooling, differences between discrete and continuous modelling can be explored. Often, equations can be impossible for any person to solve, and computers are needed to obtain mathematical and statistical properties of solutions. Examples include seemingly simple recurrence relations that generate intricate fractal images, or how to model the weather or airflow around an aeroplane.

9. Data ethics and statistical literacy are needed in the modern world.

Mathematics and statistics have developed to what they are today from human curiosity and the desire to explain and understand the world around us. Data are created, and the measurement and collection of data is an important part of any statistical inquiry. Central to this is understanding context and ensuring the data are measuring the right process and are of sufficient quality to answer the inquiry. Statistical literacy[63] is essential for critiquing the validity of knowledge gained from statistical inquiry, and ethical questions arise in the application of such knowledge.

[63] Gal, I. (2002). Adults' statistical literacy: Meanings, components, responsibilities (with discussion and a rejoinder by the author). *International Statistical Review* 70(1): 1–51.

Data are created from social, economic, and environmental situations, and biases do arise in how data are used – inherent human bias can carry through to statistical findings. Further bias can arise when the statistical findings from one population are transferred to a different population. Understanding how data are created, curated, accessed, and used, and any potential biases and ethical concerns, is increasingly important as society's expectation of data-based evidence grows. With the continual emergence of new types and sources of data, such as, digital-tracking data, and our interaction with data-based evidence, data ethics is now a fundamental part of mathematics and statistics.

One of the main sources of big data is information automatically recorded by computer systems, such as traffic or purchasing decisions and the effectiveness of advertising. Algorithms, and the data they are trained on, can also have inherent biases, and can influence our behaviours and decisions, even when we are not aware of them. Inherent biases can arise in how data are created and accessed. Scrutinising the credibility of model- and data-based evidence, conclusions, and decisions, and understanding data sovereignty, is becoming an essential requirement for full participation in our democratic society.[64]

10. Algebra and geometry are two sides of the same coin; this idea is developed via a working knowledge of useful mathematical objects.

Mathematical systems, ideas, and applications form an interconnected body of knowledge and practices that are the result of collective creative effort by many generations of humans from all cultures. This legacy includes a varied collection of abstract objects, characterised by their properties, relationships, and rules for manipulating and transforming them. For example, ancient astronomy and some navigation systems are understood via spherical trigonometry. This material may be too difficult for most learners in the early school years, but the idea of a circle has a low threshold for entry. Describing circles mathematically leads to an algebraic equation, and examining how the symmetries of a circle relate to this equation leads directly to the trigonometric functions. For another concrete case, two different plane reflections applied in succession result in a rotation; this idea can be seen by manipulating actual physical objects and understood both geometrically and algebraically. Such ideas can be learned on a variety of levels, from the basic to a complete understanding of the algebra of the underlying transformation groups. Many abstract mathematical objects are linked by geometry and algebra and these linkages are crucial to their utility.

Mathematics learning in school should include traditional geometric shapes and solids in two and three dimensions (including their description and classification via attributes and calculation of perimeters/angles/areas/volumes); coordinate systems for describing locations and transformations of objects in space; the geometry and algebra of straight lines and their intersections; and mathematical descriptions of directed and undirected networks. At higher levels of schooling, the concept of differential equations relating the changes of a system over time, or changes in one variable relative to another can be introduced and elementary examples examined for their algebraic and geometric properties.

[64] Cleveland, L., Hall, P., & Jeffers, K. (2016). IPMUS-International: Data resource for demystifying comparative statistics about society. In J. Engel (Ed.), *Promoting understanding of statistics about society.* Proceedings of the Roundtable Conference of the International Association of Statistics Education (IASE), July 2016, Berlin, Germany.

Most of these ideas involve abstract objects that can be used to encode relationships in space and/or time. The attached geometry and algebra allow them to be used to solve problems in many contexts.

Learners should be given the opportunity to explore abstract systems and geometries in other contexts, for example thinking about how to encode the choreography in dance, or devise sequences of plays in sport (where not all combinations of moves will work or have good effect!).

11. The future of mathematics and statistics requires computational thinking.

Three trends in the future of mathematics and statistics are clear:

i. the growth in computational resources, both in the capacity of computers to handle and process data, and in the expansion of computer-intensive methods in mathematics and statistics

ii. the exponential growth in available data, and the rapid change in access and type of available data (big data)

iii. the increase in interest, understanding, and place of artificial intelligence (AI) in society.

Advancing technology is driving change in the way that mathematics and statistics are done. Computer systems empower people to think about and learn from data in new ways, and earlier in their development. For large datasets the use of technology is vital. Some systems generate such complex behaviour that advanced computing power, such as supercomputers, is needed to understand them. Real-time and big data has created opportunities for knowledge extraction, but the datasets are so large that this work can be accomplished only by large computer networks. Advanced computational methods are needed to extract and manipulate data from a wide range of digital sources, including image, text, spatial, sounds, and human interactions through networks such as social media. The role of artificial intelligence in society is a trend that seems likely to accelerate in the immediate future. The impact of this will be in both what and how we teach.

Mathematics and statistics can convert a problem to be solved into a description of tasks a computer can solve algorithmically. Students should have access to authentic age-appropriate tasks that develop computational thinking. Such thinking includes identifying when a computational approach is warranted, formulating problems in a way that a computer can solve them, decomposing problems to create step-by-step instructions that can be automated, logically organising and analysing data,[65] [66] and evaluating the effectiveness and efficiency of algorithms. Coding can be used as part of statistical investigations, supporting students to think statistically with data.

[65] Budde, L., Frischemeier, D., Biehler, R., Fleischer, Y., Gerstenberger, D., Podworny, S., & Schulte, C. (2020). Data science education in secondary school: How to develop statistical reasoning when exploring data using CODAP. In P. Arnold (Ed.), T*he Proceedings of the 2020 IASE Roundtable on New Skills in the Changing World of Statistics Education*. Voorborg, The Netherlands: International Association for Statistics Education.

[66] Keller, S.A., Shipp, S.S., Schroeder, A.D., & Korkmaz, G. (2020). Doing data science: A framework and case study. *Harvard Data Science Review* 2(1). https://doi.org/10.1162/99608f92.2d83f7f5

Ngā Pūkenga Pāngarau me te Mātauranga e Tika ana Kia Mōhio ngā Ākonga, Hei Āwhea Hoki

The Mathematics Skills and Knowledge Learners Need to Know and by When

The issues the Panel identified from international comparative studies and the NMSSA report are, in part, issues of achievement level (what learners can do) and, in part, of progress. Two things need to change to address these issues: the 'amount' of mathematics children know and can do at key points in their schooling, and the rate at which ākonga students move through the curriculum.

There are two corresponding policy approaches to improving the situation: increase expectations of 'what by when' and accelerate progress. Having examined the curriculum as it stands, and the data gathered by international and national testing regimes, the Panel believes that the challenge facing our mathematics education system is not that our curriculum sets out inadequate goals, but that our students are currently not reaching them – a problem of progress, not curriculum. While we are recommending some realignment of early curriculum levels to increase our expectations of young learners in number and algebra in particular (Recommendation 3), the later levels, particularly Level 4, remain useful. The key to 'what by when' is in fully achieving what is in the curriculum at the rate of one level every two years. If achieved, by the end of Year 8 almost all our ākonga students will have attained the full breadth and depth of knowledge in Level 4 of the Aotearoa New Zealand Curriculum. Our recommendations outline the actions we think necessary to achieve this, in particular the design of appropriate assessment tools to track progress (Recommendation 7), the provision of clear resources for teaching and learning (Recommendation 8), sustained, long-term professional learning for teachers (Recommendation 9), and clarity around opportunities to learn mathematics for each year (Recommendation 3).

Ngā Arowhai o te Huarahi Mātauranga Pāngarau me te Tauanga

Checkpoints in the Mathematics and Statistics Education Pathway

Assessment is a key part of any education system. One of the most clearly established tenets of education is that knowing what students understand and can do in a way that easily feeds into teaching decisions is central to improving outcomes for learners.[67 68 69] The role of assessment is to provide sound information for good decision making. Thus, the information needs to be accurate, timely, and insightful, easily obtained and analysed, and easily interpreted and used. It also needs to align with curriculum objectives and the way that mathematics and statistics is planned and taught.

There is also a growing understanding of which type of assessment improves ākonga student learning. Large-scale assessments, like all assessments, are designed for a specific purpose. Those used to rank schools and/or ākonga students for the purposes of accountability are poor instruments for improving teaching or modifying their approach to individual students. Large-scale assessments are often taken at the end of the school year when instruction is near completion, and the results are not received until after the students have moved on. Also, the results that teachers receive usually lack the detail needed to target specific improvements.[70 71] By contrast, low stakes informal and formative quizzes and other assessments administered on a regular basis are better suited to guide improvements in student learning. Teachers trust the results because of their direct relation to classroom teaching goals, and the results are immediate and easy to analyse at the individual student level and provide feedforward for ākonga students. Assessment is an integral part of the teaching and learning process and seen as crucial for helping ākonga students learn. As significantly lower levels of anxiety are experienced in an online quiz, as compared to a traditional test, and since both reinforce learning,[72] there are various approaches to assessment which have value.

Another key use of assessment information is to evaluate the impact of teaching decisions.[73] If teaching is not resulting in learning and progress, teaching needs to change. Without sound and useful assessment information, evaluating the effectiveness of teaching approaches is not possible.

[67] Absolum, M., Fockton, L., Hattie, J., Hipkins, R., & Reid, I. (2009). Directions for Assessment in New Zealand (DANZ). 45 pages.

[68] Alton-Lee, A (2003). Quality Teaching for Diverse Students in Schooling: Best Evidence Synthesis Iteration https://www.educationcounts.govt.nz/publications/series/2515/5959

[69] Anthony, G., & Walshaw, M. (2007). Effective Pedagogy in Pāngarau/Mathematics: Best Evidence Synthesis Iteration (BES). https://www.educationcounts.govt.nz/publications/series/2515/5951

[70] Barton, P.E. (2002). *Staying on course in education reform.* Princeton, NJ: Statistics & Research Division, Policy Information Center, Educational Testing Service.

[71] Kifer, E. (2001). *Large-scale assessment: Dimensions, dilemmas, and policies.* Thousand Oaks, CA: Corwin Press.

[72] Riegel, K., & Evans, T. (2021). Student achievement emotions: Examining the role of frequent online assessment. *Australasian Journal of Educational Technology* 75-87. https://doi.org/10.14742/ajet.6516

[73] Earl, L., & Timperley, H. (2015). Evaluative thinking for successful educational innovation. OECD Education Working Papers, No.122, OECD Publishing. http://dx.doi.org/ 10.1787/5jrxtk1jtdwf-en

Clarifying how we conceptualise progress in mathematics and statistics in Years 0–10 will help harness the power of collecting and using information about learner progress to improve outcomes. The language and ideas used in the curriculum, supplementary materials, and assessments must be consistent. Aligning these three elements will reduce the amount of work teachers, students and parents must do 'to sort out how everything fits together'. Teachers will be able to clearly describe what sufficient progress is for their class and be able to plan and teach lessons that challenge learners and increase their understanding. An agreed picture of learner progress in Years 0–10, with shared language, will help the whole system communicate and work together.

High quality assessment tools are understandable and relatable outside the classroom as well. An NWEA-Gallup survey[74] in America found that 60% of parents said their child's teachers 'rarely' or 'never' discuss their child's assessment results with them, while teachers reported that they felt comfortable with most aspects of assessments – apart from "communicating with parents about the results". There is some very recent interesting Aotearoa New Zealand data here[75] in the context of students in the first year of schooling, suggesting the situation is better here in New Zealand. However, communicating clearly with parents and whānau about learner progress and achievement and communicating with learners themselves is important. Mathematics and statistics vocabulary and education-specific terms can prevent effective communication to learners and their whānau; this needs to be carefully considered when developing tools and resources so that useful information can be easily shared to support all learners.

[74] Bobowski, K. (2018). 10 ways for teachers and parents to communicate better about assessments. https://www.nwea.org/blog/2018/10-ways-for-teachers-parents-to-communicate-better-about-assessments/

[75] E. Lawes, C. Wylie, M. Alansari, M. Berg and H. Visser, Maternal perceptions of aspects of their partnerships with early learning services and schools Influences and associations. (2021) *New Zealand Council for Educational Research Rangahau Mātauranga o Aotearoa*

Ngā tūtohutanga

Recommendations

1. Convene a group of experts in Māori-medium mathematics and statistics to discuss the current state of pāngarau education, collate evidence, and make independent recommendations to the Ministry of Education (see page 11).
2. Ensure every child is given a quality opportunity to learn mathematics and statistics for at least an hour every day, Years 0–10, and that they are given access to all areas of the curriculum.
3. Clarify what ākonga students need to be given the opportunity to learn in each year in Years 0–10.
4. Move some concepts earlier in the curriculum because ākonga students can learn these things earlier and it gives them a stronger foundation for making adequate progress.
5. Cease use of within or across-class 'ability grouping' to ensure equitable learning opportunities for ākonga students and maximise the amount of quality teaching in primary and intermediate mathematics and statistical learning.
6. Revise the curriculum regularly, having an independent expert panel of mathematicians, statisticians, educators, and others to oversee its development and direction.
7. Align and clarify the language used to describe ākonga student progress in Years 0–10 and design fit-for-purpose assessment tools that provide meaningful feedback to teachers, ākonga students, and whānau families and support learning.
8. Ensure that teachers in all schools kura have equitable access to a suite of high-quality resources to support teaching at each of Years 0–13.
9. Provide research and evidence-based, sustained professional learning on mathematics and statistics knowledge for teaching, for all teachers of Years 0–8 and non-specialist teachers in Years 9–13, and consider compulsory mathematics and statistics professional learning in the induction period for provisionally certificated primary school teachers.
10. Provide research and evidence-based professional learning for teachers in Years 0–13 to support the use of inclusive, culturally sustaining pedagogies that communicate high expectations of learners and enable access for all.
11. Develop specific, named career pathways with associated remuneration, genuine time allowance, and financial support for teachers in all sectors who pursue further study or qualifications in mathematics and statistics education.
12. Investigate and implement ways to recruit more qualified teachers of mathematics and statistics in secondary schools, with targeted pathways for Māori and Pāsifika teachers.
13. Recognise that this is a whole-of-system challenge; provide clear leadership and direction for mathematics and statistics teaching and learning in Aotearoa that is accessible to all schools and kura, and improve funding for research into mathematics and statistics education to analyse what research data are available to identify the issues; and provide evidence to underpin change.
14. Provide mathematics and statistics learning opportunities for communities and whānau of school-aged learners as part of a national education campaign.

RECOMMENDATION 1.

Convene a group of experts in Māori-medium mathematics and statistics to discuss the current state of pāngarau education, collate evidence, and make independent recommendations to the Ministry of Education (see page 11).

RECOMMENDATION 2.

Ensure every child is given a quality opportunity to learn mathematics and statistics for at least an hour every day, Years 0–10, and that they are given access to all areas of the curriculum.

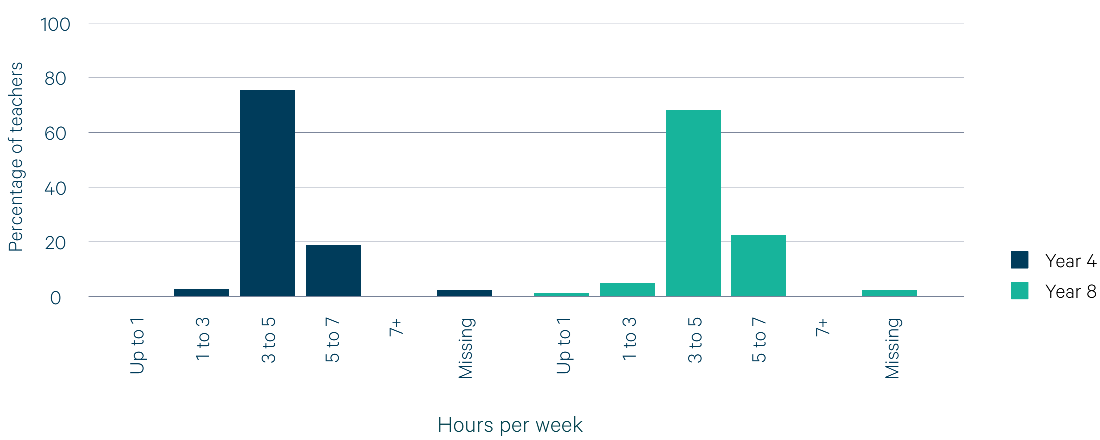
The 2018 results from the NMSSA asked teachers how much time they spent teaching mathematics each week. The results show that the majority of Year 4 and Year 8 teachers surveyed provide between three and five hours of mathematics teaching each week, with around 20% of students receiving between five and seven hours a week:[76]

Aotearoa New Zealand's widening gap between low and high scorers on the TIMSS measures, as highlighted above, may be shaped by these differential opportunities to learn offered to students.

Providing equitable learning opportunities frames other interventions and opens space for improvements in teaching and learning. Opportunity to learn describes the conditions for learning provided by teachers, which are shaped by schools and the wider education system.[77] It has several dimensions: the time allocated to learning, the value of the content being learned, and the way in which the opportunity to learn is structured. Time allocated sets the outside boundaries for what it is possible to learn. While mathematics and statistics can and should be integrated with

FIGURE 6

Percentage of teachers' responses regarding the amount of time students spend learning mathematics, on average, by year level



[76] NMSSA Report 19, Ministry of Education, 2018, p.41.

[77] Elliot, S.N., & Bartlett, B.J. (2016), *Opportunity to Learn.* Oxford Handbooks Online. Oxford University Press. https://www.oxfordhandbooks.com/view/10.1093/oxfordhb/9780199935291.001.0001/oxfordhb-9780199935291-e-70.

other learning areas, setting aside focused learning time each day will provide space for increased engagement with mathematics and statistics ideas and skills. Quality opportunities to learn centre on material that is both authentic and challenging, and relatable and attainable, for ākonga students, and are structured in ways that make the mathematics and statistics ideas and skills accessible to all. Without guaranteeing sufficient time to learn, and quality learning opportunities during that time, other interventions are hamstrung.

RECOMMENDATION 3.

Clarify what ākonga students need to be given the opportunity to learn in each year in Years 0–10.

Across mathematics and statistics teaching and learning in New Zealand we have severe slippage in relation to actual student achievement and the curriculum level at which they should be achieving. For example, the NMSSA[78] shows that while 19% of Year 4 students are working below the appropriate curriculum level in mathematics, this grows to 55% of Year 8 students. Thus, there is a lack of opportunities for students to engage with mathematical tasks at their year level. The current curriculum review should make clear what mathematics ākonga students need to cover in each year of schooling. This includes being specific about coverage of all areas of the curriculum and outlining which mathematical practices and processes ākonga students should engage with at each level.

In primary schooling in New Zealand, there has been a strong focus on teaching numeracy and the number strand of the mathematics curriculum. While none of the mathematics learning strands are optional in the New Zealand curriculum, there is choice to teach these at different times and in different years. In practice, unless schools have a clear and focused plan to ensure curriculum coverage, students moving between teachers and classrooms are not all being given the opportunity to learn across the entire mathematics and statistics curriculum. This is evidenced in TIMSS (2018) where, for example, at Year 5 only 44% of students had the opportunity to compare or draw angles and 55% engaged with parallel and perpendicular lines. Similarly, in Year 9 only 15% of students were given tasks involving solution strategies for simultaneous equations and 31% were given tasks with simple linear equations.

To ensure that ākonga students in New Zealand are given the opportunity to learn appropriate mathematics and statistics content in each year of schooling, further clarification is needed in terms of key concepts, mathematical practices, and tasks associated with learning objectives. In the current curriculum, mathematical and statistical practices/processes – what ākonga are doing as they learn mathematics and statistics – are implicit within the problem-solving focus. This contrasts with curricula from other comparable countries (such as Australia and the USA) where processes and practices are explicit. Likewise, New Zealand should state clear expectations that young learners be supported in engaging with mathematical and statistical reasoning and thinking (for example, making conjectures and developing explanations, justifications, and generalisations). That is, the design of the curriculum should signal a sequence of concepts, ways of reasoning, and strategies that ākonga employ when learning within and across content areas.

[78] Educational Assessment Research Unit, University of Otago and New Zealand Council for Educational Research for the Ministry of Education. (2019). *NMSSA 2018: Mathematics and Statistics.* Wellington: Ministry of Education.

In terms of equity, engaging in mathematical and statistical practices supports learners in developing both conceptual understanding of mathematics and statistics, and productive dispositions and identities as knowers and doers of mathematics.[79] However, as noted in PISA 2012[80] "teaching students how to question, make connections and predictions, conceptualise and model complex problems – requires time and is more challenging in disadvantaged schools". Deficit teacher expectations and out-of-field mathematics teachers mean that opportunities to learn are less likely to include support for mathematical and statistical thinking. Considerable teacher support will be needed to ensure that teaching mathematical and statistical practices does not become overly prescriptive (such as 'problem solving Fridays').

In an analysis of curriculum content, we need to consider the alignment between the formal curriculum as written, the curriculum materials accessible to teachers, and teacher practice.[81] As McChesney[82] highlights, in recent years we have had multiple proxy curricula in the primary years for mathematics and statistics in New Zealand including the Numeracy Development Project (NDP) and National Standards. A refresh of the curriculum should move away from the strategy stage approach and strong focus on counting in the early years of school promoted by the NDP. Likewise, the separation of knowledge and strategy content in NDP has promoted inconsistencies in teaching approaches, and tensions and confusion around the development of procedural fluency as it relates to practice routines for number bonds and tables (that is, basic facts) and the later recall of key formula. This tension is further fuelled for teachers (and media/parents) through the perception that high mathematical performance for Asian education systems is due to widespread rote learning.

In supporting fluency development, we need more attention and research on the nature of practice tasks. For example, drawing on successful practices in Asian countries, Marton and Tsui[83] and supported by Watson[84] note that practice routines focused on what varies and what stays the same provide "opportunities for students to observe regularities and differences, develop expectations, make comparisons, have surprises, test, adapt, and confirm their conjectures with the exercise" (p. 109). Number Talks[85] are an example of recent task development that utilise variation theory to advance number sense, fluency, and working memory, and at the same time engage learners in open creative mathematics. These types of activities support the procedural fluency necessary for students to succeed.

Giving greater clarity to the mathematical opportunities required across the curriculum for different year groups will work to reverse the slippage Aotearoa is currently experiencing. Additionally, other recommendations in this report (specifically 7, 8, and 9) support the connected approach

[79] Selling, S.K. (2016). Making mathematical practices explicit in urban middle and high school mathematics classrooms. *Journal for Research in Mathematics Education* 47(5): 505–551.

[80] OECD. (2016). *Equations and inequalities: Making mathematics accessible to all.* Paris: OECD Publishing.

[81] Schmidt, W.H., & Prawat, R.S. (2006). Curriculum coherence and national control of education: Issue or non-issue? *Journal of Curriculum Studies* 38(6): 641–658.

[82] McChesney, J. (2017). Searching the New Zealand curriculum landscape for clarity and coherence: Some tensions in Mathematics and Statistics. *Curriculum Matters* 13: 115–131.

[83] Marton, F., & Tsui, A. (2004). *Classroom discourse and the space of learning.* London: Lawrence Erlbaum Publishing.

[84] Watson, A. (2017). Pedagogy of variations: Synthesis of various notions of variation pedagogy. In *Teaching and Learning Mathematics through Variation* (pp. 85-103). Leiden: Brill Sense.

[85] Sun, K.L., Baldinger, E.E., & Humphreys, C. (2018). Number talks: Gateway to sense making. *The Mathematics Teacher* 112(1): 48–54.

required to provide appropriate learning opportunities for students. While the learning progression framework[86] goes some way toward addressing our concerns – giving some 'big picture' views of progress – it is too broad and diverse, and not directly linked to years or levels. The Panel feels teachers need the support of more detail about what to teach and when, rather than less detail and more flexibility. Creating such a resource is far beyond the remit of this panel and requires an independent group of experts with high aspirations for student attainment and good pedagogical knowledge.

RECOMMENDATION 4.

Move some concepts earlier in the curriculum because ākonga students can learn these things earlier and it gives them a stronger foundation for making adequate progress.

All children enter school with a great deal of informal or intuitive knowledge of mathematics[87] that can serve as the basis for developing the formal mathematics of the primary school curriculum. However, a number of the expectations for Number and Algebra outlined in the New Zealand curriculum document are too low for students. For example, at Level One of the curriculum (aligned with Year 1 and 2 of schooling), a strong emphasis is on counting and working with small numbers (up to 100). It is argued that "this drawn-out focus on counting strategies may substantially delay children's progress and prevent opportunities for richer mathematical learning at a young age".[88] Other mathematics education researchers[89 90 91] argue that rather than a heavy emphasis on counting, students will benefit from a variety of mathematical experiences such as subitising, comparing, and a focus on pattern and structure. These studies have highlighted how young students can abstract and generalise mathematical ideas earlier and in more complex ways than might be expected.

To ensure more effective progress in mathematics and statistics, we suggest that the curriculum refresh considers what we know about children's learning from national and international research studies and moves specific concepts earlier in the curriculum. For example, a bank of research[92 93 94] shows the benefits of developing early algebraic reasoning through generalised arithmetic from a young age. This includes providing students with opportunities to learn about equivalence and the equals sign and to investigate the properties of operations (such as commutative, associative, distributive) and numbers (such as odd and even, identity elements). Indeed, early understanding

[86] https://curriculumprogresstools.education.govt.nz/lpf-tool/

[87] Perry, B. (2013). Reflecting on young children's mathematics learning. In L. English & J. Mulligan (eds), *Reconceptualizing early mathematics learning: Advances in mathematics education.* Dordrecht: Springer.

[88] McChesney, J. (2017).

[89] Mulligan J.T., & Dockett, S. (2013) Early awareness of mathematical pattern and structure. In English & Mulligan.

[90] Hunter, J., & Miller, J. (2020). Using a culturally responsive approach to encourage early algebraic reasoning with diverse young learners. *International Journal of Science and Mathematics Education* (online first).

[91] Young-Loveridge, J. (2011). Rethinking the role of counting in mathematics learning. *Teachers and Curriculum* 12: 79–83.

[92] Blanton, M., Stroud, R., Stephens, A., et al. (2019). Does early algebra matter? The effectiveness of an early algebra intervention in Grades 3 to 5. *American Educational Research Journal* 56(5): 1930–1972.

[93] Carpenter, T., Franke, M., & Levi L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth: Heinemann.

[94] Carraher, D., Schliemann, A., Brizuela, B., Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education* 37(2): 87–115.

of the equals sign as a relational symbol indicating sameness or a relationship between two sides of an equation is a powerful predictor of later algebraic competence.[95] We argue that a focus on patterns, structure, and generalisation should be included from the beginning of the curriculum.

We also argue for a stronger focus and earlier emphasis on rational numbers. Studies[96 97 98] have shown that earlier rational number knowledge in primary schooling predicts mathematical proficiency and achievement in secondary school. It is also important to consider the instructional sequence for rational numbers in relation to whether it is introduced as fractions, decimals, and percentages, as it is currently presented within the New Zealand Curriculum. One potentially productive model from Canada[99] is to develop decimal understanding by the introduction of percentages first. Specifically, our recommendation is to have more focus on fractions at Level One of the curriculum and earlier introduction of percentages and decimals from Year 4 onwards.

Finally, we recommend the explicit inclusion of spatial reasoning from Level One upwards of the curriculum to provide ākonga students with a strong foundation for making progress in mathematics. Developing spatial thinking involves three key areas: awareness of space (including distance and dimensions), representing spatial information (in the mind and externally through diagrams, maps), and reasoning using spatial information.[100] Spatial thinking has been under-emphasised in the New Zealand Curriculum, yet there is compelling evidence that spatial competencies are predictive of mathematical achievement, and they are consistently linked to success in STEM and career achievement. [101 102 103]

[95] Matthews, P.G. and Fuchs, L.S. (2020). Keys to the gate? Equal sign knowledge at second grade predicts fourth-grade algebra competence. *Child Development* 91: e14-e28. https://doi.org/10.1111/cdev.13144

[96] Booth, J.L., Newton, K.J., & Twiss-Garrity, L.K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology* 118(1): 110–118.

[97] DeWolf, M., Bassok, M., & Holyoak, K. J. (2015a). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology* 133: 72–84.

[98] Siegler, R., Duncan, G., Davis-Kean, P., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science* 23(7): 691–697.

[99] Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education* 30(2): 122.

[100] National Research Council (2006). *Learning to think spatially: GIS as a support system in the K–12 curriculum.* Washington DC: National Academy Press.

[101] Kell, H. J., Lubinski, D., Benbow, C. P., & Steiger, J. H. (2013). Creativity and technical innovation spatial ability's unique role. *Psychological Science* 24(9): 1831–1836.

[102] Hawes, Z., Tepylo, D., & Moss, J. (2015). Developing spatial thinking: Implications for early mathematics education. In B. Davis & Spatial Reasoning Study Group (Eds.), *Spatial reasoning in the early years: Principles, assertions and speculations* (pp. 29–44). New York: Routledge.

[103] Uttal, D.H., Meadow, N.G., Tipton, E., Hand, L.L., Alden, A.R., Warren, C., & Newcombe, N.S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin* 139(2): 352–402.

RECOMMENDATION 5.

Cease use of within or across-class 'ability grouping' to ensure equitable learning opportunities for ākonga students and maximise the amount of quality teaching in primary and intermediate mathematical and statistical learning.

Evidence from TIMSS, PISA, and NMSSA highlight that primary schooling Aotearoa New Zealand, in comparison to other countries, has the highest proportion of mathematics students (86%) who work in same 'ability' groups in half their lessons or more. Similarly, a survey of 102 primary teachers[104] suggested widespread use of 'ability' grouping for mathematics teaching. Yet ability and attainment are quite different things, and the effectiveness of grouping by attainment level as a teaching strategy for mathematics is contested. A meta-analysis[105] demonstrated that grouping students by attainment only has a small likelihood of positive impacts on achievement. Other international studies[106 107] have argued some small benefits for students in attainment grouping for those students placed in the top group or stream or a small percentage identified as gifted and talented. However, these moderate benefits are outweighed by the growing evidence both in New Zealand and internationally of the harmful effects of ability grouping on both student achievement and disposition.

In-class groupings of students with similar levels of attainment limits face-to-face teaching and increases workload on teachers through preparation – some of which are implicated in our section on slippage. Further harmful effects include limited learning opportunities for high-level mathematics for students placed in middle or low attainment groups, disproportionate numbers of ethnically diverse or lower socioeconomic status students assigned to lower attainment groups, negative impact on self-confidence and learner identity, and a widening achievement gap.[108 109 110] A recent PISA study noted that the degree of a school system's vertical stratification (use of attainment grouping) was negatively related to equity of outcomes[111] – a problem already of real concern in New Zealand.

A move to not using within-class or across-class 'ability grouping' within primary schooling is a significant shift in practice, and there are many additional barriers to overcome in order to support a move towards effective teaching practices delivering more equitable outcomes. Addressing this recommendation will require specific support for teachers, and this needs to be in conjunction with the entire set of recommendations. A key step in this process will be the continued development

[104] Anthony, G., & Hunter, R. (2017). Grouping practices in New Zealand mathematics classrooms: Where are we at and where should we be? *New Zealand Journal of Educational Studies* 52(1): 73–92

[105] Hattie, J.A.C., & Yates, G.C.R. (2013). *Visible learning and the science of how we learn*. New York: Routledge.

[106] Parsons, S., & Hallam, S. (2014). The impact of streaming on attainment at age seven: Evidence from the Millennium Cohort Study. *Oxford Review of Education* 40(5): 567–589.

[107] Steenbergen-Hu, S., Makel, M.C., & Olszewski-Kubilius, P. (2016). What one hundred years of research says about the effects of ability grouping and acceleration on K–12 students' academic achievement: Findings of two second-order meta-analyses. *Review of Educational Research* 86(4): 849–899.

[108] Archer, L., Francis, B., Miller, S., Taylor, B., Tereshchenko, A., Mazenod, A., Pepper, D., & Travers, M.-C. (2018). The symbolic violence of setting: A bourdieusian analysis of mixed methods data on secondary students' views about setting. *British Educational Research Journal* 44(1): 119–140.

[109] Braddock, J., & Slavin, R. (1995). Why ability grouping must end: Achieving excellence and equity in American education. In H. Pool & J. Page (Eds.), *Beyond tracking: Finding success in inclusive schools* (pp. 7–20). Bloomington: Phi Delta Kappa Educational Foundation.

[110] Rubie-Davies, C. (2014). *Becoming a high expectation teacher: Raising the bar*. Hoboken: Routledge.

[111] Schleicher, A. (2014). Equity, excellence and inclusiveness in education: Policy lessons from around the world. Report prepared for the Fourth International Summit on the Teaching Profession. Paris: OECD.

of examples of exemplary best practice through research and evidence. The recommendations that are key to supporting this include Recommendations 2, 3, 8, 9, and 10 and we emphasise that intensive professional learning is required to support teachers to shift practice away from in-class or across-school attainment grouping.

RECOMMENDATION 6.

Revise the curriculum regularly, having an independent expert panel of mathematicians, statisticians, educators, and others to oversee its development and direction.

Currently there is no clear process or timetable for revisions of the official New Zealand Curriculum of which the mathematics and statistics learning area is a critical part. This is particularly problematic in fast-moving areas such as statistics currently is, and that computational thinking and machine learning ideas soon will be. The curriculum must not simply address current learning areas but anticipate future ones so as not to be too far behind when the lag in development and dissemination is accounted for.

Without doubt, there will soon be extremely sophisticated tools and technology supported by machine learning and AI that are likely to have an impact far more profound on learning than the introduction of calculators ever was. Who knows where the digitally advanced young people of today will take things? Our curriculum developers should be thinking about this now in order to seamlessly introduce them into future curricula. With the rapid development of 21st century knowledge, the opportunities to integrate across different learning areas must be enhanced. In particular, truly authentic applications of mathematics and statistics are now possible and should be adopted. Leadership is necessary to ensure changes to curriculum are informed by sophisticated analysis of new knowledge, and its impact on curriculum design, to support skills for jobs that may not exist for a decade.

Major and infrequent curriculum revisions lead to significant teacher workload needed to upskill content and pedagogical knowledge rapidly. It is important that resourcing for any change is in place in a timely manner to support teachers in this (see Recommendations 8, 9, 12, and 13).

The establishment of an expert panel to oversee the development and direction of regular curriculum revisions would ensure that the curriculum is more responsive to new knowledge about mathematics and statistics learning and teaching, and developments in technologies. The expert panel, through leadership from the Ministry of Education, should follow a partnership model and include Māori and Pacific educators, teachers, academics from a range of disciplines, representatives of the non-university tertiary sector, employers, and parents.

RECOMMENDATION 7.

Align and clarify the language used to describe ākonga student progress in Years 0–10 and design fit-for-purpose assessment tools that provide meaningful feedback to teachers, ākonga students, and whānau families and support learning.

Over the last two decades there have been several waves of curriculum reform and professional learning initiatives in the mathematics and statistics space in primary schools, sometimes extending to Years 9 and 10 of secondary school. The Numeracy Project, asTTLe, the New Zealand Curriculum, the National Standards, ALiM, and the Learning Progression Framework have all defined progress in mathematics in related but slightly different terms. At present, teachers juggle ‘levels’ on ‘strands’ (and 'sub levels' of beginning, proficient, advanced in some cases) (New Zealand Curriculum), 'stages' on 'domains' (Number Framework/Numeracy Project), and 'signposts' on 'aspects' (PaCT). Schools' approaches are still haunted by the National Standards language (below, at, or above standard) and the ideas behind standards-based annual assessment and reporting activities.

This must be cleaned up. While there is some alignment among these resources in underpinning research about student progression in mathematics and key ideas, there is a barrier to clear understanding and communication created by the various terms and the different approaches they embody. This is exacerbated by the tools available for assessment, each of which aligns with a different framework of progress.

Clarity and consistency in language and ideas in assessment resources must additionally extend to and integrate with that of the written curriculum and supplementary teaching materials. This will streamline the process teachers use to modify lesson plans based on sound assessment information that progresses learners appropriately through the mathematics and statistics curriculum. Similarly, quality assessment information will be easy to share with learners, among teachers and with whānau, so that the whole system communicates and works together to improve learner outcomes.

RECOMMENDATION 8.

Ensure that teachers in all schools kura have equitable access to a suite of high-quality resources to support teaching at each of Years 0–13.

The widely accepted definition of mathematics proficiency[112] includes five interrelated strands:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations.
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence: the ability to formulate, represent, and solve mathematical problems.
- Adaptive reasoning: ability for logical thought, reflection, explanation, and justification.
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Understanding the world involves tasks that support learners as they investigate, represent, and connect mathematical and statistical ideas through discussions in the context of problem solving. Problem-solving tasks "initiate learning through an appropriate challenge, effectively differentiating that challenge for particular student needs, and consolidating the learning through task variations".[113] Given the opportunities afforded by tasks determine how ākonga students come to comprehend what it means to learn and understand mathematics and statistics, task quality and enactment are key. Fit-for-purpose engagement in mathematical and statistical activities should assist students to develop:

- the ability to think creatively, critically, and logically

[112] National Research Council (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academy Press.

[113] Sullivan, P., Borcek, C., Walker, N., & Rennie, M. (2016). Exploring a structure for mathematics lessons that initiate learning by activating cognition on challenging tasks. *The Journal of Mathematical Behavior* 41: 159–170. 10.1016/j.jmathb.2015.12.002

- the ability to structure and organise
- the ability to process information
- an enjoyment of intellectual challenge
- the skills to interpret and critically evaluate statistical information in a variety of contexts
- the skills to solve problems that help them to investigate and understand the world.[114]

Problem-solving contexts can be an opportunity for students to mathematise contexts that connect to their lived experiences. Finding out about and bringing into mathematics and statistics the culture of their students – engaging in culturally sustaining pedagogy[115] – enables teachers to choose, craft, and launch tasks that engage ākonga students with big ideas in meaningful and relevant ways. However, careful thought needs to be applied to context choice and implementation to avoid tokenistic or inappropriate use.[116] Examples from Hunter and Miller[117] of links between home and school contexts for Pāsifika students in New Zealand focused on growing patterns to develop algebraic tasks. Recognising the strengths of student diversity has seen increased interest in tasks that invite students to engage in multi-dimensional ways.[118] Open tasks involve opportunities to engage in different ways, using multiple ways of representing mathematical and statistical ideas, and expressing understanding. These tasks provide teachers with opportunities to listen and respond to student thinking, and to assess formatively as the lesson progresses.

Problem-solving activities can be completed individually and through group work. With moves to more flexible grouping and culturally sustaining pedagogy, teachers need to be supported with exemplars of group-worthy tasks that enable each ākonga to have meaningful and rich learning experiences of their own and together with peers.[119] Assigning tasks to be worked on in groups without ensuring that a task can support collaborative learning will only exacerbate entrenched stereotypes about who can be smart. Group tasks need to be carefully chosen to avoid well-known challenges of unequal student participation and interactions whereby one student takes over the task while others are excluded.[120]

To support teachers to engage with all of these strands, we recommend the following resources are made available to all teachers:

- tasks that are mathematically and statistically rich, and support meaningful learning

[114] Cobb, P., & Hodge, L.L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical thinking and learning* 4(2-3): 249–284.

[115] Milner, H.R. (2011). Culturally relevant pedagogy in a diverse urban classroom. *The Urban Review* 43(1): 66–89.

[116] Clarke, D., & Roche, A. (2017). Using contextualized tasks to engage students in meaningful and worthwhile mathematics learning. *Journal of Mathematical Behaviour* 51: 95.

[117] Hunter, J., & Miller, J. (2020). Using a culturally responsive approach to encourage early algebraic reasoning with diverse young learners. *International Journal of Science and Mathematics Education* (online first).

[118] Vale, I., Pimentel, T., Cabrita, I., Barbosa, A., & Fonseca, L. (2012, July). Pattern problem solving tasks as a mean to foster creativity in mathematics. In *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, No. 1, pp. 171–178).

[119] Crespo, S. (2020). Learning to pose collaborative mathematics problems with secondary prospective teachers. *International Journal of Educational Research* 102: 101430.

[120] Anthony, G., Hunter, R., & Hunter, J. (2018). Challenging teachers' perceptions of student capability through professional development: A telling case. *Professional Development in Education* 44(5): 650–662.

[121] Kerr, B., & Averill, R. (2021). Contextualising assessment within Aotearoa New Zealand: Drawing from mātauranga Māori. *AlterNative.*

- engaging activities that support the learning of basic facts, general procedural fluency, and the development of computational fluency
- supplementary material that outlines the best ways to introduce and work with key mathematics and statistics ideas, explanations of the mathematics and statistics ideas themselves, links to existing resources, what to look for when working with ākonga, and suggestions for assessment of the key ideas
- tasks that are contextually appropriate for diverse learners in Aotearoa New Zealand, and support culturally sustaining pedagogy and assessment.[121]

The Panel recognises that some of these resources already exist, but there is confusion within schools as to the best tool for which job, and localised selection processes are too ad hoc. Commercially produced resources are only available to communities that can afford them. There also continues to be a need for authentic tools exhibiting the power of mathematical and statistical ideas in 'real world' situations such as engineering, environment, climate, social justice, and so forth.

RECOMMENDATION 9.

Provide research and evidence-based, sustained professional learning on mathematics and statistics knowledge for teaching, for all teachers of Years 0–8 and non-specialist teachers in Years 9–13, and consider compulsory mathematics and statistics professional learning in the induction period for provisionally certificated primary school teachers.

As highlighted above in the teacher discipline and pedagogical theme, there is a real and compelling need to support and improve teachers' mathematics and statistics knowledge for teaching. Debate remains about whether having knowledge of mathematics and statistics for teaching is enough in itself to make change for learners.[104] Those who see knowledge of mathematics and statistics for teaching as dynamic and situated suggest that what matters is 'knowing to'.[122 123] That is, being able to use knowledge of mathematics and statistics with learners is where the intersection occurs between teacher knowledge and learner outcomes. In addition, both experimental and large-scale quantitative studies suggest that, while it makes sense to separate out content and pedagogical knowledge in order to understand what impacts teaching and learning, in individual people the knowledge is fundamentally intertwined[124 125] and it is best learned together.[126 127] In the specific context of statistics education in Aotearoa New Zealand, the studies

[122] Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics* 38(1): 135–161.

[123] Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education* 8: 255–281.

[124] Charalambos, Y., Hill, H., Chin, M., & McGinn, D. (2020). Mathematical content knowledge and knowledge for teaching: Exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education* 23(6): 579–613.

[125] Tröbst, S., Kleickmann, T., Heinze, A., Bernholt, A., Rink, R., & Kunter, M. (2018). Teacher knowledge experiment: Testing mechanisms underlying the formation of preservice elementary school teachers' pedagogical content knowledge concerning fractions and fractional arithmetic. *Journal of Educational Psychology* 110(8): 1049.

[126] Murray, A.L., Booth, T., Eisner, M., Ribeaud, D., McKenzie, K., & Murray, G. (2019). An analysis of response shifts in teacher reports associated with the use of a universal school-based intervention to reduce externalising behaviour, *Prevention Science* 20: 1265–1273.

[127] Norton, L. (2019). Action research in learning and teaching: A practical guide to conducting pedagogical research in universities. *Psychology Learning and Teaching* 18(3).

[128] Hipkins, R., Doing research that matters: A success story from statistics education. NZCER Ministry of Education ISBN: 978-1-927231-41-8

of Pfannkuch, Arnold, & Wild and Pfannkuch, Forbes, Harraway, Budgett, & Wild, through their dissemination of findings and resources through multiple teacher networks and PLD opportunities, were identified as a success story[128] and exemplified attributes of `scaled up' professional learning as advocated in[138]. Therefore, providing research-based professional learning on mathematics and statistics knowledge for teaching in a way that deliberately supports teachers use of that knowledge with ākonga students, is most likely to improve learning outcomes.

This professional learning should recognise the scale of learning and change required by committing to long-term investment in people. All Years 0–8 teachers and non-specialist teachers of mathematics and statistics in Years 9–13 need access to sustainable, long-term opportunities to increase their mathematics and statistics knowledge for teaching and use it in practice. Likewise, the whole preparation period for new teachers (attaining a teaching qualification and two years' induction and mentoring while in a teaching role) should be considered for the development of mathematics and statistics knowledge for teaching. This should include initiatives for provisionally certificated primary school teachers that is linked to classroom practice.[129] Together, these will be powerful investments towards improving mathematics and statistics outcomes for ākonga students.

In light of this compelling need to improve teachers' mathematics and statistics knowledge for teaching, the Panel is deeply concerned that universities and other teacher education providers have cut back the mathematical provision in their Education degrees for teachers since 2005, by as much as 50%. This would seem to exacerbate the situation. The Panel suggests the Ministry of Education investigate why this has happened and the Teaching Council's role in facilitating it, and take steps to remediate the issue.

RECOMMENDATION 10.

Provide research and evidence-based professional learning for teachers in Years 0–13 to support the use of inclusive, culturally sustaining pedagogies that communicate high expectations of learners and enable access for all.

Currently, we know that opportunities to learn mathematics and statistics for our ākonga students are varied in nature and quality. For mathematics in particular, the numeracy development project (NDP) promotion of 'ability' grouping/teaching (commonly referred to as streaming)[130] has assisted deficit theorising around school readiness[131] and teacher expectations.[132 133] Promoted as a way to provide targeted support, ability grouping explicitly or implicitly created different levels of expectations for tasks and products for groups of students. The grouping by 'NDP stages'

[129] Robinson, V. &Timperley, H., (2007) The leadership of the improvement of teaching and learning: lessons from initiatives with positive outcome for students, *Australian Journal of Education*, Vol. 51, No. 3, 2007, 247–262. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.892.9172&rep=rep1&type=pdf

[130] Anthony, G., & Hunter, R. (2017). Grouping practices in New Zealand mathematics classrooms: Where are we at and where should we be? *New Zealand Journal of Educational Studies* 52(1): 73–92.

[131] Anthony, G., Hunter, R., & Hunter, J. (2018). Challenging teachers' perceptions of student capability through professional development: A telling case. *Professional Development in Education* 44(5): 650–662.

[132] Bills, T., & Hunter, R. (2015). *The role of cultural capital in creating equity for Pasifika learners in mathematics.* Paper presented at the 38th annual conference of the Mathematics Education Research Group of Australasia, Sunshine Coast.

[133] Turner, H., Rubie-Davies, C.M., & Webber, M. (2015). Teacher expectations, ethnicity and the achievement gap. *New Zealand Journal of Educational Studies* 50(1): 55–69.

promoted teaching computation in lock-step progressions, with children in lower stage groups receiving limited opportunities to progress or move out of these groupings. The diagnostic testing tools that were designed to inform teacher planning are instead more frequently used as a tool to label children; the viewpoint that "some students have more to contribute and are expected to contribute more than others while others are less capable and have less to contribute"[134] became a self-fulfilling prophecy. In those classrooms that follow the NDP scripted lesson plans, a focus on isolated computational strategies (such as compensation, tidy numbers, grouping by tens and ones) limits ākonga student opportunity to progress. As a consequence, rather than building computational fluency and confidence with early number work, these learners struggled to know which and when to apply strategies to solve number problems (see Recommendation 5 for discussion).

While the NDP provided some useful insights into learning progressions for early number skills, implementation was short on support for effective pedagogy, including that which is culturally sustainable. In terms of equity, we expect that all ākonga students will have an opportunity to develop a positive and productive relationship with mathematics and statistics.[135] In that regard, it is important that we provide Years 0–13 learners with opportunities to engage in genuine mathematics and statistics activity that is aligned with their personal values and cultural space. This will only be realised through a transformation to the way in which we teach and learn[136] that not only addresses persistent levels of underachievement in terms of content, but also in terms of wellbeing.[137]

To achieve this, professional learning and development needs to include culturally responsive/sustaining pedagogy and assessment that takes an asset-based approach[138] with learners' language and culture positioned as intellectual resource. In Aotearoa New Zealand, The Developing Mathematics Inquiry Communities (DMIC) professional development and learning programme[139] is one example that incorporates inclusive, culturally sustaining pedagogy at its heart. A key principle is that teachers are supported to provide mathematical challenges[140] that encourage ākonga students to engage in productive struggle and take risks in a trusting classroom community. In the study[141] models for developing and analysing culturally responsive teaching in mathematics teacher education were discussed and explored the perceptions of preservice and beginning teachers regarding the incorporation of cultural approaches in the teaching of mathematics.

[134] Bannister, N.A. (2016). Breaking the spell of differentiated instruction through equity pedagogy and teacher community. *Cultural Studies of Science Education* 11(2): 335–347.

[135] Hunter, R., & Anthony, G. (2011). Forging mathematical relationships in inquiry-based classrooms with Pasifika students. *Journal of Urban Mathematics Education* 4(1): 98–119.

[136] Askew, M. (2016). *Transforming primary mathematics: Understanding classroom tasks, tools and talk*. London: Routledge.

[137] Berryman, M., Lawrence, D., & Lamont, R. (2018). Cultural relationships for responsive pedagogy: A bicultural mana orite perspective. *Set 1*: 3–10.

[138] Celedòn-Pattichis, S., Peters, S., Borden, L.., Males, J., Pape, S., Chapman, O., Clements, D., & Leonard, J. (2018). Asset-based approaches to equitable mathematics education research and practice. *Journal for Research in Mathematics Education* 49(4): 373–389

[139] Hunter, R., Hunter, J., Anthony, G., & McChesney, K. (2018). Developing mathematical inquiry communities: Enacting culturally responsive, culturally sustaining, ambitious mathematics teaching. *SET: Research Information for Teachers* 2: 25-32.

[140] Sullivan, P., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Russo, J. (2020). Ways that relentless consistency and task variation contribute to teacher and student mathematics learning. In *For the Learning of Mathematics: Proceedings of a symposium on learning in honour of Laurinda Brown: Monograph 1* (pp. 32-37).

[141] Averill, R., Anderson, D., Easton, H., Te Maro, P., Smith D., and Hynds, A., (2009), Culturally Responsive Teaching of Mathematics: Three Models from Linked Studies, *Journal for Research in Mathematics Education Volume 40*.

However, as we see from both these studies, developing culturally responsive/sustaining practices in mathematics classrooms is challenging – particularly as many teachers have not experienced these types of mathematics learning environments as learners themselves. A focus on disrupting deficit thinking and promoting equity, social justice, and transforming mathematics lessons to link to ākonga students' cultures and lives involves both in-class mentoring and professional learning days. Alongside the school leadership, teacher, and student partnerships, asset-based approaches require educators to work with students, families, and communities to identify the strengths they bring to the mathematics classroom and then enact pedagogy that builds on these strengths.

Moving the provision of widescale ongoing professional learning experiences forward requires more researched understandings about local and international models of professional learning (such as Continuous Professional Development[142] and Learning Labs).[143] Rather than looking to quick fixes and methods that connect poorly with the complexities of pedagogical practices, we need to look critically at both 'how to support' and 'how to scale' reforms.[144] In light of our pressing concerns around equity, it is imperative that care be taken to develop professional learning experiences that are both adaptive and responsive to local communities.

RECOMMENDATION 11.

Develop specific, named career pathways with associated remuneration, genuine time allowance, and financial support for teachers in all sectors who pursue further study or qualifications in mathematics and statistics education.

Teacher professional learning does not end with initial teacher education but extends throughout a career. Universities offer a range of postgraduate courses and qualifications in mathematics and statistics education designed for in-service teachers, including micro credentials. These provide opportunities for teachers to engage with national and international research, to reconnect with their own identities as mathematicians and statisticians, and to examine and reflect on their professional practice. University study can support professional learning for:

- out-of-field teachers: teachers who have been recruited to teach mathematics and statistics despite specialising in other subjects (see Recommendation 12 for discussion)
- future leaders: teachers who are innovating in mathematics and statistics teaching, and who can develop and support change in schools and curriculum
- all teachers wishing to continue developing their teacher knowledge in practice.

Teachers who have engaged in university study report a lift in energy that comes from having time to reflect on their practice, to develop resources for teaching, to reconnect with or establish a passion for mathematics and statistics, and to participate in a supportive community of other professionals.[145]

[142] Leong, Y. H., Toh, T. L., Tay, E. G., Quek, K. S., Toh, P. C., & Jaguthsing, D. (2020). Scaling up of continual professional development for mathematics problem solving in Singapore schools. *International Journal of Science and Mathematics Education*. 10.1007/s10763-020-10097-3.

[143] Kazemi, E., Ghousseini, H., Cordero-Siy, E., Prough, S., McVicar, E., & Resnick, A. F. (2021). Supporting teacher learning about argumentation through adaptive, school-based professional development. *ZDM–Mathematics Education*: 1-14.

[144] Maass, K., Cobb, P., Krainer, K., & Potari, D. (2019). Different ways to implement innovative teaching approaches at scale. *Educational Studies in Mathematics* 102(3): 303–318.

[145] Barton, B., & Paterson, J. (2013). Does mathematics enhance teaching? Does summer hiking tone winter thighs? *Canadian Journal of Science, Mathematics and Technology Education* 13(2): 198–212.

University study can help teachers extend their pedagogical knowledge in topics that are notoriously difficult to teach (such as algebra, numeracy, statistics, calculus) and learn about new curriculum content and pedagogical innovations, such as computational thinking, modelling, and the role of technology in teaching and learning. Exploring school mathematics and statistics curriculum content in a university setting deepens understanding of learners' ways of thinking and how you can respond as a teacher.[146] Meetings and workshops organised by professional networks provide opportunities to share classroom practice. Sustained programmes of postgraduate study can 'graduate' primary teachers from generalists to specialists in mathematics and statistics education.[147] These teachers can then act as 'research brokers' who contribute to school support, changes in mathematics and statistics education, and the national critique and formation of educational policy.

Currently, however, there is very little incentive for teachers to pursue further study. Those wishing to become pedagogical curriculum specialists face limited career pathways,[148] with middle leadership often the only possibility. These leadership roles, however, do not explicitly incentivise further study, nor are there any requirements or incentives for out-of-field teachers to train to become in-field teachers. Greater financial incentives are available in senior management roles, and teachers will often opt to study educational leadership rather than mathematics and statistics education as a means of career progression. Curriculum expertise is completely removed from the current salary scheme, which only incentivises teachers to move out of the classroom into school leadership positions, if they do not leave teaching altogether.

We recommend developing specific named career pathways, with associated remuneration for teachers who pursue further study and qualifications in mathematics and statistics education with a view to becoming curriculum leaders in their schools. These pathways **must also provide appropriate support for teachers to pursue such study, in the form of genuine time release, stipend, and costs for professional learning**. Such a move will recognise and value teachers who develop their curriculum expertise, and who can provide expert leadership within their schools.

Finally, we point out that recruitment of mathematics teachers remains a challenge internationally, but other countries are incentivising better. For example, the base salary of a secondary teacher with 15 years' experience in our nearest neighbour Australia[149] (in 2018/19) is nearly 30% more than an equivalent New Zealand teacher ($USD 65,028 vs $USD 50,967) – a significant gap. Together with additional benefits offered in Australia, this deliberate recruitment strategy for hard-to-fill disciplines is an additional challenge for us.

[146] Wasserman, N. H. (2018). Knowledge of nonlocal mathematics for teaching. *The Journal of Mathematical Behavior* 49: 116-128.

[147] Russo, J. A. (2020). The experiences and identity structures of teacher-researcher hybrid professionals in a primary school mathematics context. *Eurasia Journal of Mathematics, Science and Technology Education* 16(7): em1861. https://doi.org/10.29333/ejmste/8250

[148] In other countries, there are different pathways (and associated renumeration) for teachers wishing to specialise in curriculum, distinct from leadership. For example, see Singapore's three teaching tracks: https://www.moe.gov.sg/careers/become-teachers/pri-sec-jc-ci/professional-development

[149] OECD (2019). https://data.oecd.org/teachers/teachers-salaries.htm

RECOMMENDATION 12.

Investigate and implement ways to recruit more qualified teachers of mathematics and statistics in secondary schools, with targeted pathways for Māori and Pacific peoples.

Aotearoa New Zealand has experienced a long-standing issue with recruiting teachers of mathematics and statistics in secondary school, and is failing to attract sufficient teachers to fill teacher supply shortages and address declining student achievement. The Ministry of Education's incentives such as scholarships, including tuition fees and a stipend, has failed to increase enrolment numbers (~160) of prospective secondary teachers of mathematics and statistics and related STEM subjects from 2011 to 2017[150]

Recently regulations for Initial Teacher Education (ITE) programmes have changed and require only that each provider sets a minimum subject knowledge for prospective secondary teachers.[151] This is far less prescriptive than previously where, to specialise in mathematics and statistics, teaching candidates needed to have completed undergraduate mathematics and statistics study at 200-level for those aiming to teach junior secondary (Years 9 and 10) and 300-level for those aiming to teach senior secondary (Years 11, 12, and 13). To specialise in mathematics and statistics teaching, candidates should have high levels of mathematical knowledge. Graduates with such qualifications, however, are highly desirable in business, finance, marketing, and STEM industries, where lucrative career pathways are open to them. The teaching profession competes with a range of employment sectors for these graduates, but with less career advancement potential and financial reward (see last paragraph of the previous recommendation). A compounding problem is that even industry cannot recruit the number of mathematics and statistics graduates they require, making competition all the more fierce.

As identified in Recommendation 11, many schools cope by enlisting secondary teachers who have specialised in another field and primary teachers into Year 9 classrooms, many of whom go on to teach Years 10 and 11 without sufficient professional learning. Data from TIMMS shows that 37% of Year 9 students in New Zealand are taught mathematics and statistics by a teacher without discipline specialisation. This shortage is longstanding;[152] 15 years earlier, data from the 2004 teacher census showed 25% of secondary school mathematics teachers had no tertiary mathematics qualifications.

While the shortage of mathematics and statistics specialist teachers is felt nationwide, it is particularly acute in rural and low socioeconomic schools, in Māori-medium, and Māori and Pacific peoples teachers. Increasing the numbers of Māori and Pacific peoples teachers of mathematics and statistics is critical for supporting Māori and Pacific peoples students as they may better understand the needs, values and ways of learning for Māori and Pacific peoples students and are role models who provide visible pathways to follow. Their presence creates an immediate sense of community for Māori and Pacific peoples students, and they help to shape cultural awareness for all staff in schools through their actions and contributions.

[150] Ministry of Education. (2019). *Initial teacher education statistics*. Retrieved from https://www.educationcounts.govt.nz/statistics/initial-teacher-education-statistics

[151] *ITE programme approval, monitoring and review requirements* (2019). https://teachingcouncil.nz/resource-centre/research-and-development/development-of-the-ite-programme-requirements/

[152] Anthony, G., Butler, P., & Rawlins, P. (2011). Secondary mathematics and statistics teachers: Who are our teachers of the future? *New Zealand Mathematics Magazine* 48(1): 23–37; Ministry of Education. (2005).

Previous incentives and new ideas should be critically evaluated and appropriately implemented to meet the current demand for graduates. Efforts should be made to look beyond mathematics and statistics graduates alone to include other disciplines such as data science, computer science, artificial intelligence, mathematical biology, science, technology, engineering, and economics. These graduates have significant knowledge of mathematics and statistics applications that will, as discussed in Recommendation 9, enrich mathematics and statistics teaching in secondary school. For many of these disciplines, mathematics and statistics are embedded in 300-level courses rather than taught separately, requiring flexibility from the Teaching Council of Aotearoa New Zealand regarding disciplinary requirements for admission into Teacher Education and certification programmes. This flexibility should be supported by extensive PLD, so candidates gain essential disciplinary and pedagogical knowledge if needed.

RECOMMENDATION 13.

Recognise that this is a whole-of-system challenge; provide clear leadership and direction for mathematics and statistics teaching and learning in Aotearoa that is accessible to all schools and kura, and improve funding for research into mathematics and statistics education to analyse what research data are available to identify the issues; and provide evidence to underpin change.

Currently there is a whole-of-system challenge in building capability and capacity that derives from:

- no system-wide approach to developing mathematics and statistics education leaders
- a proliferation of school PLD providers that are independent of ITE providers and each other, which contributes to a lack of coherence in emphasis and focus across the system[153]
- lack of a 'middle-layer' or coherent infrastructure to support teacher learning[154]
- a separation of research and PLD, which risks the provision of advice that is not evidence-based and not aligned to system level evaluation and assessment such as those reported by international agencies like OECD.[155]

System structures need to be purposely designed to support the development and sustainability of effective mathematics and statistics education. It is important that the framing of specific suggestions for addressing the whole-of-system challenge for mathematics and statistics are seen in the context of the New Zealand education system. Strengthening the support infrastructure will be critical to the alignment and coherence in the system. The following suggestions should be considered:

- establish and support system-wide networks of external expertise
- develop a Mathematics and Statistics Education Institute to support a system-level network of external expertise for schools

[153] O'Neil, J. (2011). The privatisation of public schooling in New Zealand. *Journal of Education* 26: 17–31;
Wylie, C. (2012). *Vital Connections. Why we need more than self-managing schools*. Wellington: NZCER Press.

[154] Wylie (2012).

[155] Nusche, D., Laveault, D., MacBeath, J., & Santiago, P. (2011). OECD Reviews of evaluation and assessment in education: New Zealand. *Policy* 26(1), 17-31. Paris: OECD.

- increase post-graduate mathematics and statistics education study opportunities for teachers through fees subsidies and teacher scholarships
- strengthen open access equitable dissemination systems for teachers and policy developers
- use mathematics and statistics leadership and networks to develop and evaluate resources for schools.

RECOMMENDATION 14.

Provide mathematics and statistics learning opportunities for communities and whānau of school-aged learners as part of a national education campaign.

Mathematics is often a curriculum subject parents recall in a negative light and make disparaging comments about their own abilities, like 'I was never any good at mathematics'.[156] These types of comments contribute to poor achievement in mathematics being seen as acceptable. However, we know that effective partnerships between parents and schools can result in better outcomes for students.[157] Currently, in New Zealand and internationally, little is known of how families interact in ways related to STEM outside of the school setting. Even less is known about everyday practices related to mathematics of families from non-dominant communities.[158] [159] We do know, however, that teachers position diverse students and their families within deficit perspectives[160] (see Recommendation 10).

We can look both in New Zealand and internationally to interrogate productive parent and community engagement initiatives that promote mathematics and statistics learning opportunities and develop whānau and school partnerships. A key theoretical framework that underpins much of the work on parent and community engagement is 'funds of knowledge'. This is underpinned by the belief that all people and cultures have bodies of knowledge and skills that are historically accumulated, culturally developed, and support individual/household functioning and wellbeing.[161] Developing asset-based approaches means acknowledging, valuing, and building upon the funds of knowledge all participants hold. These parent and community engagement initiatives could be part of a nationwide campaign to promote mathematics and statistics education in communities.

[156] Muir, T. (2009). At home with numeracy: Empowering parents to be active participants in their child's numeracy development. In R. Hunter, B. Bicknell, & T. Burgess (Eds.) *Crossing divides (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia*, pp. 395-402). Wellington, NZ: MERGA.

[157] Mutch, C., & Collins, S.K. (2012). Partners in learning: Schools' engagement with parents, families, and communities in New Zealand. *School Community Journal* 22: 167–187.

[158] Civil, M. (2016). STEM learning research through a funds of knowledge lens. *Cultural Studies of Science Education* 11(1): 41–59.

[159] Williams, J. J., Tunks, J., Gonzalez-Carriedo, R., Faulkenberry, E., Middlemiss, W. (2016). Supporting mathematics understanding through funds of knowledge. *Urban Education* (Advance online publication).

[160] Turner, H., Rubie-Davies, C.M., & Webber, M. (2015). Teacher expectations, ethnicity and the achievement gap. *New Zealand Journal of Educational Studies* 50(1): 55–69.

[161] Civil, M., & Hunter, R. (2015). Participation of non-dominant students in argumentation in the mathematics classroom. Intercultural Journal, 26(4), 296-312. https://directorymathsed.net/public/Ireland/IrelandPapersPDF&Docx/Civil%20&%20Hunter%20(4).pdf

I orea te tuatara ka patu ki waho

A problem is solved by continuing to find solutions

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Print ISBN 978-1-877317-49-1
Digital ISBN 978-1-877317-50-7

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